THE CROSS-FLOW DRAG ON A MANOEUVRING SHIP

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Abstract—In this paper, an analysis is given of the experimentally derived local lateral force on a manoeuvring ship as a reaction to the ship's lateral velocity. The tests were performed with models consisting of several segments. Special attention is paid to the longitudinal distribution of the non-linear component of the lateral force, the so-called cross-flow drag. This aspect is of utmost importance when the non-linear contribution becomes dominant as will occur in a tight turn during which the ship's drift velocity becomes relatively large compared to the ahead speed.

NOMENCLATURE

AP  aft perpendicular
Cl  lift coefficient
Cd  drag coefficient
Ccp  corrected drag coefficient
Fn  Froude number = \( U/(g \cdot L_{pp})^{0.5} \)
FP  forward perpendicular
G  ship's center of gravity
g  acceleration due to gravity
L  lift on the ship = force perpendicular to the resultant speed \( U \) of the ship
L_{pp}  ship's length
l_n  length of the \( n \)th segment
m_y  added mass in lateral direction per unit length
r  ship's rate of turning
T  ship's draught
l_n  draught of the \( n \)th segment
u  ship's speed component in the longitudinal direction
U  resultant speed of ship = \( (u^2 + v^2)^{0.5} \)
v  ship's speed component in the lateral direction
x  longitudinal direction in the ship's fixed system of coordinates
x_e  longitudinal direction in the Earth fixed system of coordinates
y_e  lateral direction in the Earth fixed system of coordinates
Y  lateral force to the ship
N  yawing moment around \( G \)
N_e  yawing moment around the midship section
\( \beta \)  drift angle = \( \text{atan}(\nu/u) \)
\( \rho \)  mass density of the water.

1. INTRODUCTION

For the prediction of the ship's manoeuvrability, use can be made of computer simulations. For this approach mathematical models must be available by which the relevant hydrodynamic forces on the manoeuvring ship are described.

Often these mathematical descriptions of the hydrodynamic phenomena have been derived from a regression analysis of the results of captive model tests for the ship considered [see Hooft (1986)]. However, such mathematical descriptions are not always...
satisfactory because of the lack of a physical meaning of the descriptions. This imperfection may lead to the following problems:

- The impossibility of comparing the hydrodynamic coefficients of two different ships. This problem may be serious when designing the ship for optimum manoeuvrability.
- A limited accuracy of the manoeuvring predictions because of the fact that the simulation results will depend on the accuracy by which the coefficients have been derived by the regression analysis.

For the prediction of the ship's manoeuvrability in the initial design stage, no model test results are usually available for the determination of the hydrodynamic coefficients. One can then use computer simulations only if the hydrodynamic coefficients can be established otherwise.

Nowadays it is not yet possible to determine the hydrodynamic manoeuvring coefficients by means of theoretical methods. Therefore, empirical methods have been developed by means of which, in the initial design stage, the hydrodynamic coefficients can be estimated as a function of the ship's main dimensions; see, for example, Inoue et al. (1981) and Kijima et al. (1990).

In empirical methods the linear hydrodynamic coefficients are described rather accurately as a function of only a few aspects of the ship's dimensions. Two reasons can be given for the achieved accuracy:

1. The linear hydrodynamic coefficients are most likely rather independent of local parameters of the ship's hull form (at the bow and/or the stern).
2. For a wide range of ships the linear hydrodynamic coefficients have been determined experimentally. This means that an acceptable level of confidence has been achieved in the description of the coefficients as a function of the ship's main parameters.

It appears that the non-linear contribution of the hydrodynamic characteristics can be estimated only roughly by empirical methods. This is very inconvenient when tight turns have to be predicted by means of computer simulations. In such manoeuvres the non-linear contributions play a significant role. Three reasons are mostly given for this unfavourable aspect:

1. The non-linear hydrodynamic coefficients are rather sensitive to the local form parameters of the ship. This means that much more information (data) is required as a function of the larger number of parameters.
2. Only for a limited number of ships have non-linear hydrodynamic coefficients been determined experimentally. This means that only a limited level of confidence has been achieved in the description of the coefficients as a function of the ship's main parameters.
3. Often only the hydrodynamic coefficients have been published without the actual model test results. Some of the authors use quadratic non-linear coefficients, while others apply tertiary non-linear coefficients. In this way the validity of a presented non-linear coefficient is limited and cannot be compared with the corresponding coefficient for another hull form.

It was thought that a better description of the non-linear component of the lateral force could be achieved if one could establish the local non-linear force component
instead of the total force on the ship. This local component was defined by the local cross-flow drag coefficient; see, for example, Hooft (1987).

Burcher (1972), Clarke (1972), Matsumoto (1983) and Beukelman (1988) have presented the results of their experiments with segmented models. From these tests the lateral forces on each of the segments can be derived. In the present paper a description is given of the analysis of test results with segmented models. Based on this analysis an empirical method can be derived to estimate the distribution of the non-linear lateral force component over the ship's length. The intention of this method is:

1. To achieve a more accurate prediction of the non-linear component of the lateral force.
2. To realize a more accurate mathematical description of the hydrodynamic forces from measurements on the total model.
3. To establish eventually a simulation program that is capable of predicting the ship's manoeuvrability with sufficient accuracy. Such a manoeuvring prediction method would provide a means to investigate the effect of the variations of the ship's hull form on its manoeuvrability.

2. THE VARIATION OF THE LATERAL VELOCITY OVER THE LENGTH OF THE SHIP

While considering the ship's manoeuvring performance, three velocity components can be discerned at each instant (see Fig. 1):
• \( u \), the ahead velocity of the centre of gravity \( G \)
• \( v \), the lateral velocity of the centre of gravity
• \( r \), the rate of turning around the vertical axis.

The combination of drift velocity \( v \) and yaw rate \( r \) leads to a local lateral velocity \( v(x) \) that varies over the length of the ship:

\[
v(x) = v + x \cdot r.
\]

When considering a turn by a port (positive) rudder angle then it is seen that the ship will turn to port (negative \( r \)) at a positive drift velocity \( v \). In this turn the local

![Fig. 1. Definition of the ship-fixed system of coordinates.](image)
drift velocity becomes zero (the turning point) somewhere near the bow of the ship \((x>0)\), while at the stern the lateral velocity will become quite large relative to the ship's ahead speed. As a consequence of this result it is found that, in general, the lateral hydrodynamic reaction force at the bow will be small, while at the stern quite a large force will be experienced.

3. THE LATERAL FORCE AS A REACTION TO THE DRIFT VELOCITY

When considering first the lateral hydrodynamic force on the ship as a reaction to the ship's drift velocity \(v\) only (at zero rate of turning), the following description is often applied:

\[
Y(v) = Y_{uv} \cdot u \cdot v + Y_{vv} \cdot v^2 / v
\]  

or, in a non-dimensional form:

\[
Y(\beta)' = Y_{\beta} \cdot \cos(\beta) \cdot \sin(\beta) + Y_{\beta \beta} \cdot \sin(\beta) \cdot \sin(\beta)/\sin(\beta)
\]  

in which the lateral force has been made non-dimensional by dividing by \(0.5 \rho L_{pp}^2 U^2\) or \(0.5 \rho L_{pp} T U^2\) with the total velocity \(U\) defined by:

\[
U = (u^2 + v^2)^{0.5}.
\]

The drift angle \(\beta\) is defined by:

\[
\beta = \arctan(v/u)\;
\]

The total lateral force \(Y(v)\) is the resultant of all lateral forces \(Y_n(v)\) on each of the segments in which the ship model has been subdivided:

\[
Y(v) = \sum Y_n(v)
\]

while the total yawing moment around the center of gravity amounts to:

\[
N(v) = \sum \left(x_n \cdot Y_n(v)\right).
\]

For further analysis the local lateral force on each segment is written in a non-dimensional form according to:

\[
Y_n(\beta)' = Y_n(v)/(0.5 \rho l_n t_n U^2)
\]

with \(l_n\) the length of the segment and \(t_n\) its average draught; see, for example, Fig. 2.

According to Equation (3) one now describes the non-dimensional local lateral force \(Y_n(\beta)'\) by means of a linear coefficient \(C_{yn}\) and a non-linear coefficient \(Cd_n\):

\[
Y_n(\beta)' = C_{yn} \cdot \sin(\beta) \cdot \cos(\beta) - Cd_n \cdot \sin(\beta) \cdot \sin(\beta)/\sin(\beta)
\]

in which the linear coefficient \(C_{yn}\) corresponds to the derivative \(Y_{n\beta}\) and the drag coefficient \(Cd_n\) corresponds to \(-Y_{n\beta\beta}\).

It can be shown that in Equation (2) the linear term is linearly dependent on the longitudinal velocity component \(u\) and not on the total velocity \(U\). This means that for zero forward speed \((u = 0)\) no linear contribution in the lateral force exists. This
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holds not only for the lateral force on the total ship but also for the local lateral force
on each of the segments of a ship model.

If one considers the lift \( L \) to be a lateral force that is perpendicular to the total
undisturbed incoming flow \( U \), then the coefficient \( C_y \) is related to the local lift
coefficient \( C_l \) as follows:

\[
C_y = -C_l \cos(\beta) .
\]  

(10)

Inserting this result into Equation (9) leads to:

\[
Y_n(\beta) = -C_l \sin(\beta) \cos(\beta)^2 - C_d \sin(\beta) / \sin(\beta)/.
\]  

(11)

In the application of Equation (11), use is made of the assumption that the lift
coefficient \( C_l(n) \) is independent of the drift angle \( \beta \). This means that it is accepted
that the local cross-flow drag coefficient is a function of the drift angle \( \beta \). Various
physical arguments can be given to prove this assumption to be correct. It should be
borne in mind that both the lift coefficient and the cross-flow drag coefficient vary
over the length of the ship.

From the test results on each segment as presented in Fig. 2 one determines the
local lift coefficient \( C_l(n) \) at the range of small drift angles \( \beta \). With this value of
\( C_l(n) \) one determines for each segment the cross-flow drag coefficient from the
measurements at higher drift angles:

\[
C_d(n) = \frac{-Y_n(\beta) \cos(\beta)^2 \sin(\beta)}{\sin(\beta) / \sin(\beta)}.
\]  

(12)

From this equation one finds, in Table 1, the values of the drag coefficients \( C_d \) for
the test results in Fig. 2, which were presented by Beukelman (1988).
TABLE 2. COMPARISON BETWEEN MEASURED AND CALCULATED LINEAR LATERAL FORCE COMPONENT (IN A NON-DIMENSIONAL FORM) ON EACH OF THE SEVEN SEGMENTS OF A MODEL OF THE TODD 70 SERIES, NO TRIM, \( F_n = 0.15 \); see Beukelman (1988)

<table>
<thead>
<tr>
<th>Drift angle ( \beta ) (in degrees)</th>
<th>Measured dimensionless latitudinal force ( Y_\xi )</th>
<th>Drag coefficient ( C_{L_\beta} ) with ( C_{L_\beta}(5) = -0.287 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>+0.01478</td>
<td>1.057</td>
</tr>
<tr>
<td>8</td>
<td>+0.01906</td>
<td>1.038</td>
</tr>
<tr>
<td>12</td>
<td>+0.00806</td>
<td>1.134</td>
</tr>
<tr>
<td>16</td>
<td>-0.00398</td>
<td>1.015</td>
</tr>
<tr>
<td>20</td>
<td>-0.02001</td>
<td>0.912</td>
</tr>
</tbody>
</table>

Tod 70 model (7 segments); \( L_{pp}/T = 17.5 \); zero trim; 5th segment from the bow; \( F_n = 0.15 \).

4. THE LOCAL LIFT COEFFICIENT \( C_{L_\beta}(x) \)

According to Jones (1946), one finds that the local lift per unit length is determined by the instantaneous apparent acceleration of the local lateral added mass of water alongside the ship:

\[
L_\xi = v \cdot u \cdot m_y \xi
\]

with \( m_y \) being the lateral added mass of water per unit length of the ship and \( \xi \) being the distance of the cross-section from the forward perpendicular. In Equation (13) the derivatives \( L_\xi \) and \( m_y \xi \) are defined by:

\[
L_\xi = \frac{dL}{d\xi}; \quad m_y \xi = \frac{dm_y}{d\xi}.
\]

From integration of \( L_\xi \) in Equation (13) one finds that the lift force \( L \) over a distance between \( \xi_f \) and \( \xi_a \) (with \( \xi_f \) being the closest to the FP) amounts to:

\[
L = v \cdot u \cdot \int_{\xi_f}^{\xi_a} m_y \xi \, d\xi = v \cdot u \cdot (m_y(\xi_a) - m_y(\xi_f))
\]

from which it is seen that in theory the total lift on the ship will be zero (paradox of d'Alembert) because of the fact that in an ideal fluid the added mass per unit length \( m_y \) is zero at the bow (\( \xi_f = 0 \)) as well as at the stern (\( \xi_a = L_{pp} \)).

In Tables 2 and 3 the local lift coefficient \( C_{L_\beta}(n) \) on each of the seven segments of

\[\begin{array}{c|c|c|c|c|c|c|}
\hline
L_{pp}/T & 22.81 & 17.50 & 14.20 \\
\hline
Segment No. & \( C_{L_\beta} \) (measured) & \( C_{L_\beta} \) (calculated) & \( C_{L_\beta} \) (measured) & \( C_{L_\beta} \) (calculated) & \( C_{L_\beta} \) (measured) & \( C_{L_\beta} \) (calculated) \\
\hline
1 & +1.233 & +1.134 & +1.387 & +1.440 & +1.577 & +1.655 \\
2 & +0.301 & +0.312 & +0.401 & +0.370 & +0.487 & +0.487 \\
3 & +0.172 & +0.115 & +0.186 & +0.142 & +0.215 & +0.176 \\
4 & 0 & -0.009 & -0.040 & -0.003 & 0 & -0.001 \\
5 & -0.199 & -0.259 & -0.215 & -0.299 & -0.242 & -0.329 \\
6 & -0.172 & -0.437 & -0.287 & -0.606 & -0.356 & -0.722 \\
7 & +0.284 & -0.944 & +0.143 & -0.934 & +0.072 & -1.109 \\
\hline
\end{array}\]
the ship model is given as derived from the experiments described by Beukelman (1988). In these tables the local lift coefficients are also presented, derived from Equation (15).

From the comparison between the local lift coefficients, the theoretical values for the hull form considered agree well with the experiments for the five most forward segments.

The theory by Jones (1946) for determining the local lift force can also be checked in another way while applying Equation (15). For this purpose one reduces from the measurements the distribution of $m_y(\xi)$ by a summation of the lift derivative $L_{uv}(n)$ over the segments after Equation (15) has been rewritten by:

$$m_y(\xi_a(n)) = m_y(\xi_a(n-1)) + L_{uv}(n)$$

with $m_y$ being zero at the forward perpendicular.

In Figs 3 and 4 the experimental values of $m_y$ are presented as derived by means of Equation (16) using $L_{uv}$ from the model test results presented by Beukelman (1988). In Figs 3 and 4 the theoretical values of $m_y$ are also plotted. From the comparison of the theoretical and experimental values in these figures it is seen that the theory should be corrected for three-dimensional effects on the value of $m_y$ leading to $m_{yc}$:

$$m_{yc}(\xi) = C_m(\xi)*m_y(\xi)$$

in which $C_m(\xi)$ is the correction term as a function of the longitudinal location. Having determined the corrected lateral “added mass” distribution $m_{yc}(\xi)$ it will be possible to determine the corrected derivative $m_{yc}$:

$$m_{yc}(\xi) = m_{yc}(\xi) = dm_{yc}(\xi)/d\xi.$$.

Applying the value $m_{yc}$ in Equation (15) one now finds that the total lift on the ship amounts to:

$$L = v*u* \int_{\xi_{APP}}^{\xi_{PFP}} m_{yc} d\xi = v*u*m_{yc}(\xi = \xi_{APP}).$$

When applying the theory of Jones (1946), while using the corrected values of $m_y$,
one then finds for the total yawing moment due to the lift force distribution over the ship's length:

\[ N(\text{around the midship section}) = v \cdot u \cdot \int_{\xi_{FPP}}^{\xi_{APP}} (0.5L_{pp} - \xi) \cdot m_{\xi \xi} \cdot d\xi \]
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Fig. 4. Comparison of the calculated lateral mass $m_y$ per unit length with the values derived from the experiments with segmented models for a Todd 70 hull form with $Lpp/T = 17.5$ at $Fn = 0.15$; see Beukelman (1988).

or

$$N_0 = v \cdot u \cdot [-m_{yc}(\xi_{APP}) \cdot Lpp/2 + \int_{\xi_{FPP}}^{\xi_{APP}} m_{yc}(\xi) \, d\xi].$$  \hspace{1cm} (20)
5. THE LOCAL CROSS-FLOW DRAG COEFFICIENT $Cd(\beta)$

After having determined the lift coefficient $Cl_p(n)$ on each segment, one then establishes the cross-flow drag coefficient $Cd_n(\beta)$ on each segment by means of Equation (12). Some of the results derived in this way are presented in Fig. 5. In this figure the distribution of $Cd_n$ over the ship length has been plotted for various drift angles $\beta$.

The following comments on the results in Fig. 5 can be given:

The cross-flow drag coefficient on the first segment from the bow has most probably been caused by the bow wave. Therefore it is assumed that this value depends on the forward speed of the ship; see also the findings by Matsumoto (1983).

Aside of the cross-flow drag coefficient on the most forward segment(s) it is seen from the results in Fig. 5 that for small drift angles $\beta$ the value of $Cd$ will increase at larger distances from the bow until some maximum value is attained after which $Cd$ will decrease a little. At increasing drift angles $\beta$ this curve of $Cd$ over the ship’s length will move forward.

For large drift angles of approximately 90° the ship sails nearly abeam. In this condition the distribution of the cross-flow drag coefficient over the ship’s length mainly depends on the form of the ship’s hull and to a lesser degree on the local Reynolds number. In Fig. 6 this distribution of $Cd$ is presented for a containership and a tanker; the results in this figure follow from Matsumoto (1983).

The value of $Cd(\xi)$ at a given cross-section will vary at increasing drift angles from the values shown in Fig. 5 to the values shown in Fig. 6.

One now considers the relation between $Cd_n(\beta \neq 90^\circ)$ at an arbitrary drift angle and $Cd_n(\beta = 90^\circ)$ at a drift angle of 90° for the various ship types and ship conditions. For this purpose use is made of the corrected drag coefficient $Ccd_n(\beta)$ which is defined by:

$$C_{cd}(\beta) = \frac{Cd(\beta \neq 90)}{Cd(\beta = 90)}$$

leading to the fact that at 90° drift angle the coefficient $Ccd$ equals unity over the whole length of the ship.

Combining the results in Figs 5 and 6 according to Equation (21) will yield the results presented in Fig. 7. In Fig. 8 the course of $Ccd$ over the ship length is schematically indicated for increasing drift angles.

From the results of the corrected drag coefficient $Ccd_n(\beta)$ in Fig. 7 it is concluded that this coefficient is still dependent on the longitudinal location and the drift angle, but is no longer dependent on the ship’s hull form.

6. CONCLUSIONS

Experiments have been carried out with segmented models during which the lateral forces on each of the segments were measured. These tests were performed as a function of the ship’s lateral velocity (in a towing tank) and as a function of the ship’s rate of turning (under a rotating arm facility).

In this paper the results are presented of the analysis of the experiments with segmented models as a function of the ship’s lateral velocity. The same kind of results were found by analysing the rotating arm tests as a function of the ship’s rate of turning.
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LPP/T = 22.81
LPP/T = 17.50
LPP/T = 14.20

Fig. 5. Distribution of the cross-flow drag coefficient over the ship’s length, derived from the experiments with a segmented model for a Todd 70 hull form without trim at $Fn = 0.15$; see Beukelman (1988).
The conclusions obtained in the present study can be summarized as follows:

- Based on Jones' theory (1946), the distribution of the linear lateral force over the ship length can be predicted as a function of the ship's form by means of empirical formulations.
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\[ LPP/T = 22.81 \]
\[ LPP/T = 17.50 \]
\[ LPP/T = 14.20 \]

Fig. 7. Distribution of the corrected cross-flow drag coefficient \( C_{cd} \) over the ship's length, derived from the experiments with a segmented model for a Todd 70 hull form without trim at \( F_n = 0.15 \); see Beukelman (1988).
The distribution of the cross-flow drag coefficient can be predicted for a lateral drifting ship ($\beta = 90^\circ$) as a function of the ship's form parameters by means of empirical formulations.

With the knowledge of the drag coefficient for $90^\circ$ drifting it is possible to predict the cross-flow drag coefficient for any drift angle $\beta$ as a function of the ship's form parameters by means of empirical formulations.

The results presented in this paper show that it is possible to develop a manoeuvring prediction program that is based on a physical theory rather than on regression. Such a program leads to more accurate predictions of the ship's manoeuvrability because the effects of local hull form parameters are taken into account.

REFERENCES


