On the coupling terms in the low-frequency viscous reaction forces of moored tankers in deep water

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Abstract

Due to the restricted water depth of experimental facilities one has to rely more on computational tools for the design of turret and spread moored tanker based FPSOs in deep water. These tools have to be as accurate as possible. The mechanics to compute dynamically mooring lines and risers are understood. By means of potential theory the first order tanker motions and wave drift forces with and without current can be adequately computed nowadays, see for instance Ref. 1.

Besides the low-frequency (lf) wave drift forces also the lf viscous reaction forces (damping) exist in still water and current. The lf reaction forces/moment induced by the lf motions in the horizontal plane are mostly dominated by viscosity and hardly to compute. By means of model tests the viscous forces in the separate surge, sway and yaw modes of motion were measured in still water and in a current field, see for instance Ref. 2. The importance of the coupling terms due to the combined modes of motions, however, is not known.

By means of a lf large stroke mechanical oscillator prescribing combined motions on the tanker hull the coupling terms were investigated in still water and in current. The results will be presented in the paper.

Introduction

In order to compute the lf motions of a moored tanker the equations of the lf motion have to be established. Most of the maneuvering models are based on the model as for instance presented by Abkowitz, Ref. 3. In this model the maneuverability is described by means of linear and non-linear derivatives of the hydrodynamic forces and moments by perturbation models.

Contrary to normal maneuvering applications (high speed, high rate of turn and relative small drift angles) specific requirements have to be fulfilled for a moored tanker e.g.: large drift angles (0-360 degrees)

small values of drift velocities (combined low frequency tanker motions in still water or in a current field)

small values of oscillating rate of turn

relatively large transverse motions

in steady state in a current field the hydrodynamic forces have to correspond with the steady current loads.

These requirements do not satisfy the model of Abkowitz. The full equations of the lf motion as applied for a moored tanker in still water and in a current field should be established. A present formulation is presented. Starting in an ideal and irrotational fluid the equations of motion as given by Norrbin (Ref. 4), taking into account the Munk-moment (Ref. 5) can be established. In real fluid, however, viscosity is involved. The viscosity modifies the Munk-moment and introduces additional resistance forces (d’Alembert paradox).

By means of model tests using a lf large stroke oscillator the viscous forces in the separate surge, sway and yaw modes of motion can be determined. A comprehensive description is given in Ref. 2.

Moored tankers in general, however, perform fish tailing motions. A fish tailing motion consists of a combined motion in the horizontal plane. In the combined sway and yaw mode of motions coupling terms could exist.

In order to investigate coupling terms model tests were applied to a tanker for separate and combined sway-yaw motions using the lf large stroke oscillator. The water depth can be considered as deep water. The model tests were carried out both in still water and in a current field. In this paper the results of oscillation tests in current concern head current only.

The content of the investigation will be presented in the following sections:

- Theory still water (equations of lf motion)
- Theory in a current field (equations of lf motion)
- Test set-up oscillation tests
- Presentation of results in still water
- Presentation of results in current
- Time-domain simulations with a bow hawser moored tanker in still water exposed to a regular head wave
- Time-domain simulations with a bow hawser moored tanker in a current field and finally
Conclusions.

Theory still water

The equations of the LF motion with the hydrodynamic terms
for still water in the ship-fixed system of co-ordinates, as
derived from Ref. 2 and Ref. 6, are presented below.

\[
(M + a_{11}) \ddot{x}_1 = (M + a_{22}) \ddot{x}_2 \dot{x}_6 + a_{26} \dot{x}_6^2 + X_{1SW}
\]

\[
(M + a_{22}) \ddot{x}_2 = -(M + a_{11}) \dot{x}_1 \dot{x}_6 + X_{2SW}
\]

\[
(I + a_{66}) \dot{x}_6 + a_{62} \ddot{x}_2 = -(a_{22} - a_{11}) \dot{x}_1 \dot{x}_6 - a_{62} \dot{x}_6^2 + X_{6SW}
\]

in which the viscous terms in still water are defined as follows:

\[
X_{1SW} = -B_1 \ddot{x}_1
\]

\[
X_{2SW} = -\frac{1}{2} \rho_T \int C(l)(\dot{x}_2 + \dot{x}_6) \ddot{x}_2 \ddot{x}_6 dl
\]

\[
X_{6SW} = -\frac{1}{2} \rho_T \int C(l)(\dot{x}_2 + \dot{x}_6) \ddot{x}_6^2 dl
\]

where:

\[B_1\] = still water damping coefficient in surge direction
\[\rho\] = specific density of water
\[T\] = draft of tanker
\[AP, FP\] = distance to Aft perpendicular-CoG (negative) and
distance Fore perpendicular-CoG (positive) respectively
\[C(l)\] = Transverse resistance coefficient as function of position
along the center line of the tanker

The still water damping coefficients and the transverse
resistance coefficients were determined from oscillation tests
in still water, see section “presentation of results in still
water”.

The other symbols represent:
\[M\] = mass tanker
\[a_{11}, a_{22}, a_{62}, a_{66}, a_{26}\] = added mass in surge direction, sway
direction, yaw direction due to sway, yaw direction and sway
direction due to yaw respectively.

The added mass coefficients are determined by means of 3-D
potential theory for \(\omega \to 0\) rad/sec.

Theory in a current field

The equations of LF motion with the hydrodynamic terms in a
ship-fixed system of co-ordinates in a current field, as derived
from Ref. 2 and 6, are described below.

\[
(M + a_{11}) \ddot{x}_1 = (M + a_{22}) \ddot{x}_2 \dot{x}_6 + X_{1stat} + X_{1dyn}
\]

\[
(M + a_{22}) \ddot{x}_2 + a_{26} \dot{x}_6 = -(M + a_{11}) \ddot{x}_1 \dot{x}_6 + X_{2stat} + X_{2dyn}
\]

\[
(I + a_{66}) \dot{x}_6 + a_{62} \ddot{x}_2 = X_{6stat} + X_{6dyn}
\]

in which the static current load contributions are computed
according to:

\[
X_{1stat} = \frac{1}{2} \rho T L C_{1c} |v'_{cr}| \psi^2
\]

\[
X_{2stat} = \frac{1}{2} \rho T L C_{2c} \psi^2
\]

\[
X_{6stat} = \frac{1}{2} \rho T L C_{6c} \psi^2
\]

with:

\[L\] = length between perpendiculars
\[C_{1c}, C_{2c}, C_{6c}\] = current resistance coefficients in surge, sway
and yaw direction according to Ref. 2
\[\psi_{cr}\] = relative current angle of incidence
\[v'_{cr}\] = relative current velocity

and the dynamic current load contribution, which consist of a
potential part and a viscous part, is assumed to be:

\[
X_{1dyn} = -(a_{22} - a_{11}) V_C \sin(\psi_{cr} - \phi) \dot{x}_6 + x_{1D}
\]

\[
X_{2dyn} = -(a_{22} - a_{11}) V_C \cos(\psi_{cr} - \phi) \dot{x}_6 + x_{2D}
\]

\[
X_{6dyn} = x_{6D}
\]

in which:

\[x_{1D} = 0.6 \times (a_{22} - a_{11}) V_C \sin(\phi_{cr} - \phi) \dot{x}_6
\]

\[x_{2D} = 0.5 \times \rho \cdot L^2 \cdot T \cdot \left| X'_{2v} v_{cr} \cdot \dot{x}_6 + X'_{2v} \right| \cdot \nabla v_{cr} \cdot \dot{x}_6 \right| + X'_{2r} \cdot L \cdot \dot{x}_6^2 + X'_{2r} \cdot \nabla \cdot \dot{x}_6 \cdot \dot{x}_6 \cdot L^2 / v_{cr}
\]

\[+ X'_{2r} \cdot \nabla \cdot \dot{x}_6 \cdot L^2 / v_{cr}
\]

\[X_{6D} = 0.5 \times \rho \cdot L^3 \cdot T \cdot \left| X'_{6v} v_{cr} \cdot \dot{x}_6 + X'_{6v} \right| \cdot \nabla v_{cr} \cdot \dot{x}_6 \right| + X'_{6r} \cdot L \cdot \dot{x}_6^2 + X'_{6r} \cdot \nabla \cdot \dot{x}_6 \cdot \dot{x}_6 \cdot L^2 / v_{cr}
\]

\[+ X'_{6r} \cdot \nabla \cdot \dot{x}_6 \cdot L^2 / v_{cr}
\]

All viscous terms were derived from oscillation tests in a
current field with different current speeds. The quantities of
the coefficients are given in Ref. 2.

The added mass coefficients are determined by means of 3-D
potential theory for \(\omega \to 0\) rad/sec. No forward speed was
applied.

In order to investigate possible hydrodynamic coupling terms
in the equations of motion oscillation tests were performed for
both the still water and the current case. The test set-up will be
described in the following section.
Test set-up oscillation tests

General. A laden 200 kDWT tanker was used. The particulars are given in Table 1, while the small body plan is presented in Figure 1.

The oscillation tests were carried out in the shallow water basin of MARIN. The basin measures 220*15.8 m. The water depth was 1m. A scale of 1:82.5 was applied simulating a water depth of 82.5 m, which is considered to be deep water. To the carriage the lf large stroke oscillator was mounted. The oscillator can prescribe two combined modes of motion being a translation and a rotation motion. Both amplitudes, phase angle, periods and tanker heading can be adjusted.

The tanker was connected to the oscillator by means of a yoke construction. The yoke construction was attached to the model by means of one longitudinal and two transverse force transducers fitted in ball-jointed rods, allowing heave, pitch and roll. The transducer in longitudinal direction was more sensitive than the transducers as applied in transverse direction in order to increase the accuracy of the measurements.

The test set-up of the interface yoke-tanker is given in Figure 2. The complete test set-up is shown in Figure 3.

Still water. In still water separate sway test (x_s=30 m, T=350, 250 and 150 s), yaw tests (x_0=20°, T=350, 250 and 150 s) and combined sway and yaw tests (x_s=30 m and x_0=20° with a phase angle ε_s=0° for T=350, 250 and 150 s) were carried out. The phase angle ε_s=0° means that the sway and yaw motions were in phase. Theoretically the measured results should correspond to the following forces and moment in respectively surge, sway and yaw direction:

\[ X_1 = -(M + a_{t1}) \dot{x}_1 + (M + a_{t2}) x_2 \dot{x}_6 + a_{12} x_6^2 + X_{1SW} \]
\[ X_2 = -(M + a_{t2}) \dot{x}_2 - a_{26} x_6 - (M + a_{t1}) \dot{x}_6 + X_{2SW} \]
\[ X_6 = -(I + a_{66}) \ddot{x}_6 - a_{62} \ddot{x}_2 - (a_{22} - a_{t2}) \dot{x}_2 - a_{62} x_6 \dot{x}_6 + X_{6SW} \]

The computed and measured results of the tests are given in the section “Presentation of results in still water”.

Current. In current the oscillation tests were carried by towing the test set-up. Two current speeds were applied being 1 m/s and 2 m/s. The mean current direction was 180° (head current).

In current separate sway tests (x_s=30 m, T=350, 250 and 150 s), yaw tests (x_0=20°, T=350, 250 and 150 s) and combined sway -yaw tests (x_s=30 m and x_0=20° with a phase angle ε_s=0° for T=350, 250 and 150 s) were carried out. Theoretically the measured results should correspond to the following forces and moment in respectively surge, sway and yaw direction:

\[ X_1 = -(M + a_{t1}) \dot{x}_1 + (M + a_{t2}) x_2 \dot{x}_6 + X_{1stat} + X_{1dyn} \]
\[ X_2 = -(M + a_{t2}) \dot{x}_2 - a_{26} x_6 - (M + a_{t1}) \dot{x}_6 + X_{2stat} + X_{2dyn} \]
\[ X_6 = -(I + a_{66}) \ddot{x}_6 - a_{62} \ddot{x}_2 + X_{6stat} + X_{6dyn} \]

The computed and measured results of the tests are given in the section “Presentation of results in current”.

Definition presenting results

The results of the computations and measurements are given for the in-phase and out-of-phase force/moment components.

In-phase: the computed or measured force/moment component, which correspond to the maximum positive amplitude of sway, yaw or sway-yaw combination (=inertia part +static current load).

Out-of-phase: the computed or measured force/moment component, which corresponds to maximum positive velocity amplitude in sway, yaw or combined sway-yaw direction (=viscous part).

Mean: the measured mean refers to the average value determined over a complete measured cycle.

Ship-fixed and earth-bound system of co-ordinates: the system of co-ordinates used in the equations of motion is defined as to be ship-fixed. The system of co-ordinates is shown in Figure 2.

It must be noted that for the combined sway-yaw motions the sway acceleration of the oscillator are earth-bound. The earth bound acceleration in sway direction (in-phase) has to be composed in accelerations in the ship bound accelerations, see also Table 3.

Current direction: the (relative) current direction is defined as 180° being head current. Current entering into starboard side is 90°, while coming in from port side is denoted as 270°.

Presentation of results in still water

The results of the measurements and the computations are given in Table 3. The added mass quantities as derived from 3-D potential theory are given in Table 2.

The results of the (out-of-phase) viscous reaction force/moment components of the separate sway and yaw modes of motion as function of the square of the associated velocities are given in Figure 4. The force/moment components clearly show a quadratic relation with the applied velocities.

These results were used to determine the resistance coefficients along the centerline of the vessel. Since the Yy (=X_{22}), Ny (=X_{62}), Yn (=X_{26}) and Nn (=X_{66}) force/moment components are known, the vessel were divided in four parts to determine the associated resistance coefficients. The chosen parts were section 0-2, section 2-4, section 4-18 and section 18-20, see also Figure 5. The four unknown resistance coefficients were solved and the results are given in Figure 5. The computed resistance coefficients were used to calculate the out-of-phase force and moment components.

Reviewing the results of the combined motion tests it can be concluded that the uncoupled resistance coefficients well described the reaction force/moment component in the combined modes of motion. Except for the x-direction, it can be concluded that the comparison between the computed and measured results is very good.

While in the separate modes of motion hardly a surge force...
occurs, the measured surge force in the combined modes of motion was significant large. The X1-force has been plotted as function of the square of the associated velocities in Figure 6. A quadratic relation with the velocity is clearly shown. The computed results in x-direction, however, show peculiar or suspicious values especially in the sign.

Presentation of results in current
The results of the measurements and computations are given in Table 4, 5 and 6. The added mass quantities as used in the computations are given in Table 2

Sway mode of motion. The computed and measured results for the sway mode of motion in head current are given in Table 4. The results give a good correlation between computed and measured values.

Yaw mode of motion. In Table 5 the computed and measured results for the yaw mode of motion in head current are presented.

From the results it can be concluded that the computed and measured values reasonably match. At shorter periods the computed Y- and N-components both in-phase and out-of phase tend to deviate from the measured values. The values of the computed in-phase components increase to some extend, while the values of the out-of-phase components decrease.

The computed results in x-direction, however, show peculiar values especially in the sign.

Combined sway-yaw modes of motion. Table 6 gives the results of the combined sway and yaw motions. Comparing the computed and measured in-phase and out-of-phase components in Y- en N-direction a fairly good agreement is found. The out-of-phase components suffer the same deviation as found for the separate yaw motion: at shorter periods the values increase with the regard to the measured results. The in-phase part, however, shows a good agreement.

While in the separate modes of motion the current loads dominated the surge force, the measured surge force in the combined modes of motion increased significantly at shorter oscillation periods. For both current speeds the X1-forces have been plotted as function of the square of the associated velocities in Figure 6. A quadratic relation with the velocity is closely matched.

The computed results in x-direction, however, show peculiar values especially in the sign.

Comparison present method and approach of Obokata.
For the out-of-phase (viscous) component besides the computations with the present method also computations were carried out using the formulation of Obokata, Ref. 7.

The formulation of Obokata is given below and represents the viscous part in the earlier mentioned dynamic current load contribution.

\[ X_{1D} = 0 \]
\[ X_{2D} = 0 \]
\[ X_{6D} = \frac{1}{2} \rho \frac{T}{\Delta P} \int \left[ C_{2c} (\psi_{cr} (l)) (V_{cr} - \dot{X}_{l})^2 + u_{cr}^2 - C_{2c} (\psi_{cr}) V_{cr} \right] \text{d}l \]

in which:
\[ \psi_{cr} = \text{relative current angle of incidence at position } l \]
\[ V_{cr} = \text{relative current velocity} \]
\[ u_{cr} = \text{relative current component in surge direction at CoG} \]
\[ v_{cr} = \text{relative current component in sway direction at CoG} \]
\[ C_{2c} = \text{current resistance coefficient in sway direction} \]

The results of the computations are shown in Table 5 and 6. From the results it can be concluded:

- the X_{3D} viscous part may contribute significantly to the dynamic current load contributions and should be taken into account.
- the X_{4D} viscous part is the only dynamic part in the dynamic current load distribution. In the separate yaw test the present method underestimates the viscous yaw contribution to some extend, while the method Obokata seriously underestimates the viscous yaw contribution. In the combined modes of motions, however, the Obokata formulation presents fairly good results of the viscous yaw distribution in head current.

Time-domain simulations
General. The measured results in general present a significant coupling in surge direction in the combined modes of motions as is shown in Figure 6. Comparing the X1-force as measured in still water and current, it can be concluded that the X1-force is the largest in still water, smaller in Vc= 1 m/s and smallest in Vc= 2 m/s.

In spite of the relative small values in comparison to the reaction forces in Y- and N-direction, these small values has a considerable impact on the results of simulations. Two examples will be shown to demonstrate the effect of the X1-force on simulation results. One case is a bow hawser moored tanker in still water exposed to a regular wave. The other case is a bow hawser moored tanker decaying in a current. In both cases the hawser is moored to a fixed point in space.

Simulation in still water. The test set-up and the initial conditions of the hawser moored tanker are given in Figure 7. The tanker is the same 200 kDWT as used during this study. The hawser characteristic is given in Figure 8. The unloaded hawser length amounts to 75 m.

The vessel is exposed to a regular wave with a height 2\( \zeta_{w} = 4.72 \text{ m} \) and a period of T=7.14 seconds. The QTF of the wave drift forces as function of heading is given in Figure 9. For the computations the estimated damping in surge direction amounts to:

- Wave drift damping \( B_{1} = 98.1 \text{ kN.s.m}^{-1} \)
- Still water surge damping \( B_{11} = 121.6 \text{ kN.s.m}^{-1} \)

The computations were carried out with the present method for still water. The time-domain results are given in Figure 10. Referring to Figure 6 and taking into account a sway and yaw period with T~1000 seconds a mean X1-force=-192.2 kN was added in surge direction.

The results with and without the mean X1-force clearly show the sensitivity of the X1-force on the tanker behavior and the hawser force. By adding X1-force=-196.2 kN (in the equation of motion in surge direction) a decrease of the hawser force
from 3000 kN to 1600 kN was found. The results (with the added X1-force) correspond well with the results of model tests. The model test results are shown in Figure 11. The computations were carried out exactly as the model tests. The details of the model tests are given in Ref.2.

The time-domain simulations show the importance to know correctly the mean X1-force in surge direction. More study has to be carried out to understand this mean (coupled) X1-force and the peculiar results of the present computed in-phase and out-of-phase data.

**Simulation in current.** The test set-up of the hawser moored tanker is given in Figure 7. The tanker is the same 200 kDWT as used during this study. The hawser characteristic is given in Figure 8.

The vessel is moored in a current field. As is shown in Figure 7 the tanker was kept in position by a set of rods under an initial angle with respect to the current. By releasing the rods the tanker was free to move and the tanker performs decay in current. Two conditions were investigated being:

1) Unloaded bow hawser length 45 m, \( V_c = 1.54 \text{ m/s} \) for a X1-force=0 kN; for initial position, see Figure 12 and a X1-force=-294.3 kN; for initial position, see Figure 12
2) Unloaded bow hawser length 75 m, \( V_c = 1.03 \text{ m/s} \) for a X1-force=0 kN; for initial position, see Figure 12 and a X1-force=-196.2 kN; for initial position, see Figure 12

The computations were carried out with the present method for current. The results are given in Figure 12. The results clearly show the impact of the added X1-force on simulations. The computed results (with the added X1-force) correspond well with the results of model tests. The model test results have been given in Figure 13. Details of the model tests are given in Ref.2.

As said in the section “simulations in still water” also for the simulations in current it can be said that the simulations show the importance of the measured mean force in surge direction. More study has to be carried out to understand and predict this mean (coupled) X1-force and to understand the peculiar results of the computed in-phase and out-of-phase data.

**Conclusions**

From the results of the study the following conclusions can be drawn:

1) Still water: By means of the measured out-of-phase data as obtained from the separate sway and yaw modes of motion the distribution of the transverse resistance coefficients over the length of the vessel can be determined. These uncoupled coefficients can be well used to compute the transverse resistance of the vessel for the combined sway and yaw modes of motion.
2) Still water: The computed in-phase (inertia) and out-of-phase (viscous) of the Y- and N-components correspond very well with the results of the model tests.
3) Still water: In the separate sway and yaw test a negligible small surge force was measured. In the combined modes of motion a significant surge force was measured.
4) Head current: The computed in-phase and out-of-phase components in Y- and N-direction for both the separate and combined sway and yaw modes of motions correspond reasonably to fairly good with the results of the model test.
5) Head current: The results of the present method and the formulation of Obokata were compared. The viscous part in Y-direction contributes significantly to the dynamic current contribution and can not be neglected. In separate yaw motions the dynamic yaw contribution is seriously underestimated using Obokata. In the combined mode of motion the results are fairly good.
6) Head current: In the separate sway and yaw test a surge force was measured in the order of current loads. In the combined modes of motion a significant large (coupled) X1-force was measured.
7) Results of simulations show the importance of a correct prediction of this surge force.
8) More study is necessary to understand and predict this measured mean (coupled) X1-force and to understand the peculiar results of the computed in-phase and out-of-phase data.

**Acknowledgements**

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**References**

2. Wichers, J.E.W.:”A simulation model for a single point moored tanker”, PhD, Delft University of Technology, 1988
5. Munk, M.:”The aerodynamics of airship hulls”, NACA Report No. 184, 1924
7. Obokata, J.:”Mathematical approximation of the slow oscillation of a ship moored to single point moorings”, Marintec Offshore China Conference, Shanghai, October 1983.
### TABLE 1 – MAIN PARTICULARS OF TANKER

<table>
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<tr>
<th>Designation (Model No. 5612)</th>
<th>Symbol</th>
<th>Unit</th>
<th>Loaded</th>
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<td></td>
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<tr>
<td>Length between perpendiculars</td>
<td>Lpp</td>
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<td>Center of gravity above keel</td>
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<td>Height</td>
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### TABLE 2 – THE LOW FREQUENCY ADDED MASS QUANTITIES

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TABLE 3- SEPARATE SWAY, YAW AND COMBINED SWAY/YAW MOTIONS IN STILL WATER

<table>
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<tr>
<th>Component</th>
<th>$x_{2a}$ and $x_{6a}$</th>
<th>T</th>
<th>computed kN(m)</th>
<th>measured kN(m)</th>
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<td>m or deg</td>
<td>sec</td>
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<td>Inphase (inertia)</td>
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*) In-phase part: oscillator accelerations transform to ship-fixed accelerations: 

\[ \text{in-phase: } xdd1 = -x_{2a} \omega^2 \sin(x_{6a}) \]

(sway and yaw motions in phase $\epsilon_{26} = 0^\circ$)

\[ \text{in-phase: } xdd2 = -x_{2a} \omega^2 \cos(x_{6a}) \]

---

TABLE 4-SEPARATE SWAY MOTION IN HEAD CURRENT (180°)

<table>
<thead>
<tr>
<th>Component</th>
<th>$x_{2a}$</th>
<th>T</th>
<th>$v_x$</th>
<th>computed kN(m)</th>
<th>measured kN(m)</th>
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<tr>
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<td>m</td>
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<td>m/s</td>
<td>outphase</td>
<td>inphase</td>
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### TABLE 5—SEPARATE YAW MOTION (x6a = 20°) IN HEAD CURRENT (180°)

<table>
<thead>
<tr>
<th>T</th>
<th>v_c (m/s)</th>
<th>X1stat (180°)</th>
<th>X1D</th>
<th>Obokata computed kN(m)</th>
<th>measured kN(m)</th>
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*) potential part in y-direction: -(a22-a11)*v_c*cos(ψc-x6)*x6a

### TABLE 6—COMBINED SWAY-YAW MOTION (x2a=30 m-x6a =20°; INPHASE) IN HEAD CURRENT (180°)

<table>
<thead>
<tr>
<th>T</th>
<th>Vc (m/s)</th>
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<th>Y2aD</th>
<th>N2aD</th>
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<td>kN(m)</td>
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</table>

*) x-direction: -0.4*(a22-a11)*v_c*sin(ψc,x6)*x6a in which X1D is included; y-direction: -(a22-a11)*v_c*cos(ψc,x6)*x6a

**) (M+a22)*x2aD *x6a
Fig. 1 - Small body plan 200 kDWT tanker

Fig. 2 - Test set-up and system of co-ordinates
Fig. 3 - Low frequency large stroke oscillator for combined motions in the horizontal plane

Fig. 4 - Relation between reaction forces/moments and sway and yaw velocity in still water (positive directed velocities)
Fig. 5 - Resistance coefficients over the length of the tanker in still water (derived from sway and yaw tests)

Fig. 6 - Mean X₁ force as function of sway and yaw velocity during combined motions
Fig. 7 - Test set-up for still water and current test (Location A is 4.5 m before Fpp)

Fig. 8 – Hawser Characteristics

Fig. 9 - Computed QTF of the wave drift forces and moment as function of wave direction (2\(\zeta_a=2\) m=7.14 s)
\( \text{X}-\text{force}= 0 \text{ kN-initial conditions:} \)
\[ x(1)=-237.59 \text{ m}, x(2)=-19.96 \text{ m}, x(6)= 7.5^0 \]

\( \text{X}-\text{force}=-196.2 \text{ kN-initial conditions:} \)
\[ x(1)=-239.59 \text{ m}, x(2)=-19.96 \text{ m}, x(6)= 7.5^0 \]

Fig. 10 - Computer simulations in still water exposed a regular head wave (2\( \zeta \alpha = 4.72 \text{ m-T=7.14 s} \)) with and without X1-force
X1-force= 0 kN; initial position: x1=-201.96 m, x2=-16.86 m
X6=6.33; Vc=1.54 m/s; hawser length=45 m

X1-force= -294.3 kN; initial position: x1=-201.96 m, x2=-16.86 m
X6=6.33; Vc=1.54 m/s; hawser length=45 m

Fig. 12 - Computed yaw decay in current with and without mean X1-force

X1-force= 0 kN; initial position: x1=-229.17 m, x2=-14.92 m
X6=5.6; Vc=1.03 m/s; hawser length=75 m

X1-force= -196.2 kN; initial position: x1=-233.17 m, x2=-14.92 m
X6=5.6; Vc=1.03 m/s; hawser length=75 m

Fig. 11 - Test results of hawser moored tanker exposed to a regular head wave- 2 ζa=4.72 m- T=7.14 s (Ref. 2)
Fig. 13 – Test results of the yaw decay in current (Ref.2)