MAXIMUM LIKELIHOOD METHOD AS A MEANS TO ESTIMATE THE DIRECTIONAL WAVE SPECTRUM AND THE MEAN WAVE DRIFT FORCE ON A DYNAMICALLY POSITIONED VESSEL

Olaf J.Waals, 
Graduate student, 
TU Delft/MARIN 
2, Haagsteeg 
6700 AA Wageningen 
The Netherlands

Drs. A.B.Aalbers, 
Project Manager, 
MARIN 
2, Haagsteeg 
6700 AA Wageningen 
The Netherlands

Prof.dr.ir.J.A.Pinkster, 
Professor Ship Hydromechanics, 
TU Delft 
2, Mekelweg 
2628 CD DELFT 
The Netherlands

ABSTRACT
Feed forward in control theory is a method in which real time information about system disturbance is fed into the controller to improve its performance. As such, feed forward of the wave drift forces would improve DP behavior of a ship in terms of fuel consumption as well as position keeping. In the present study the wave drift forces have been divided in a constant part and a low frequent oscillating part. The constant part directly depends on the directional wave energy spectrum. In this paper the directional spectrum and mean drift force will be estimated from six relative wave height measurements on a dynamically positioned vessel.

The Extended Maximum Likelihood Method (EMLM) is known to make a reliable estimate of the directional wave spectrum from wave measurements at fixed locations in the wave field.

For a wave feed forward application the EMLM had to be implemented on a moving ship. Six relative wave height probes have been installed on board of a shuttle tanker. The EMLM has been applied to these relative motions and the low frequent yawing motion has been taken into account to calculate an earth bound spectral estimate. The estimate for the spectrum is based on a 30min average and is updated every minute in a moving average algorithm. Finally, the mean wave drift force is calculated for the actual heading of the ship.

Keywords: MLM, Directional Spectrum, Wave Drift Forces

INTRODUCTION
In deep water offshore activities an increasing use is made of dynamic positioning. Because of the high safety, environmental and financial risks involved in operating a DP vessel, the DP system must be designed as reliable as possible, while maintaining cost efficiency. This means the fuel consumption as well as the wear and tear of the machinery has to be minimized while on the other hand position-keeping behavior has to be improved.

One of the improvements thought feasible is to apply wave feed forward to the DP controller. By enabling the ship to anticipate its surrounding waves, the forces working on the ship can be counteracted the moment they occur. Pinkster [1] firstly introduced the idea of wave feed forward. In his work he has shown that wave feed forward can actually lead to a significant improvement in DP behavior.

scope and objectives
The objective in this paper is to reliably estimate the earth fixed directional wave spectrum from six relative wave height measurements on the vessel. From the spectrum, the mean wave drift force will be calculated for the actual heading of the ship.

In order to achieve the wave force feed forward, the sea-state has to be estimated in real time. The wave drift forces acting on the ship may be split up into two parts: a mean part and a low frequency part. In this paper only the mean part will be considered. In order to estimate the low frequency part, deterministic information of the passing wave groups would be needed. Such information is lost in the spectral analysis.

The Extended Maximum Likelihood Method has been chosen as most suitable to estimate the directional spectrum. Many authors such as Isobe [2], Benoit [4] and Jefferys [3,6],
confirm the high-resolution power of EMLM and the limited calculation time needed. These two properties are considered highly valuable in the real time DP control process.

**NOMENCLATURE**

\( \theta \) Wave direction  
\( \omega \) Wave frequency  
\( k \) Directional wave number  
\( H(\omega, \theta) \) Directional transfer function  
\( H(\omega) \) Non-directional transfer function  
\( \text{DSF}(\omega, \theta) \) Directional Spreading Function  
\( \text{CPSD}(\omega) \) Cross Power Spectral Density  
\( \Phi \) CPSD matrix  
\( n, m \) Sensor number  
\( N \) Number of directions  
\( M \) Number of frequencies  
\( \kappa \) Scaling factor  
\( \hat{\cdot} \) Denotes an estimate

**PROPOSED METHOD FOR ESTIMATION OF THE DIRECTIONAL WAVE SPECTRUM AND MEAN DRIFT FORCES**

**Measurement configuration**

The DP shuttle tanker ‘Loch Rannoch’, operated by BP Shipping, has been instrumented with six relative wave height sensors. These sensors are mounted on the bow, stern and fore- and aft- shoulders of the vessel.

The EMLM is applied to three probes at a time, because in a real sea the distance between the sets of sensors is so large that spatial aliasing corrupts the correlation between the signals (see section discussion). Figure 1 depicts the measurement configuration on the bow of the ship. The wave sensors are indicated as black marks (1, 2 and 3).

![Figure 1: Measurement configuration at the bow](image)

The Directional Spreading Function (DSF)

The calculation scheme for estimation of the Directional Spreading Function (DSF) from relative wave motions has been applied by Jefferys [2] and follows directly from the article by Isobe [3]. The Directional wave spectrum is assumed to be the product of the wave energy spectrum and the directional spreading function as discussed by Benoit [4]:

\[
S(\omega, \theta) = E(\omega) \text{DSF}(\omega, \theta)
\]

Eq 1

The total estimated power for each frequency has to stay the same and can not become negative, which is why the following conditions apply:

\[
\int_0^{2\pi} |\text{DSF}(\omega, \theta)| d\theta = 1
\]

\[
DSF(\omega, \theta) \geq 0, [0..2\pi]
\]

Eq 2

**Extended Maximum Likelihood Method**

1. Phase based EMLM:

The EMLM assumes that the DSF can be written as a linear combination of the Cross Power Spectral Densities (CPSD) between the measured signals. For each frequency all combinations of cross power spectral densities are calculated and the CPSD matrix is formed.

\[
\text{CPSD}_\omega \text{MAT} = \begin{bmatrix}
\Phi_{11} & \Phi_{12} & \Phi_{13} \\
\Phi_{21} & \Phi_{22} & \Phi_{23} \\
\Phi_{31} & \Phi_{32} & \Phi_{33}
\end{bmatrix}_{\omega}
\]

Theoretically, for each wave direction and frequency a certain combination of cross power spectral densities is expected. This expected CPSD consists of the geometrical phase difference (\( e^{i\xi} \)) and the normalized transfer-functions for relative wave height (\( |H(\omega, \theta)| \)). The EMLM minimizes the difference between the measured CPSD and the theoretical values, resulting in a frequency dependent directional estimate. According to EMLM the estimated DSF can be written as:

\[
\hat{\text{DSF}}_{\text{MLM}}(\omega, \theta) = \kappa \left( \sum_{m,n} H_m(\omega, \theta) \Phi_{mn}(\omega) H_n^*(\omega, \theta) e^{-i\xi(\phi_m - \phi_n)} \right)
\]

Eq 3

In which \( \kappa \) has been defined a constant, so that equation 2 is satisfied.
II - Amplitude based EMLM:

Since the phases of the transfer functions introduced severe aliasing (see discussion), an alternative method was investigated, using the amplitudes of the relative wave heights only. Here, use is made of the difference in amplitude between the sensors at the weather- and leeward- side of the vessel. The amplitude ratios between the signals have been used to minimize over the directions. A least squares fit has been used to minimize the measured amplitude combinations 2-3, 5-6, 3-6, 2-5 and 1-4 with respect to their theoretical counterpart. In which sensors 4, 5 and 6 on the stern correspond to sensor 1, 2 and 3 on the bow respectively. The calculation schemes for both methods are depicted in figure 3.

Estimation of total wave power

Since the absolute wave height is not measured anywhere and \( \kappa \) is chosen to satisfy equation 2, the true wave auto power spectrum has to be calculated. This is possible by calculating a non-directional transfer function \( H(\omega) \) from the directional transfer function \( H(\omega, \theta) \) and estimated DSF(\( \omega, \theta \)). This is done as follows:

\[
H_n(\omega) = \frac{2\pi}{\theta_2 - \theta_0} \int_0^{2\pi} DSF(\omega, \theta) \cdot H_n(\omega, \theta) \, d\theta
\]

Eq 4

The above directional transfer function was obtained from linear potential diffraction theory. The resulting non-directional transfer function can be applied straightforward to the measured signal. The estimated energies for all probes are averaged.

Calculation of the Cross Power Spectral Density (CPSD-) matrix

The Weighted Overlapping Segment Averaging method has been used to estimate the CPSD between the signals. It is based on the theory given by Welch[5] and averages short time Fast Fourier Transforms (FFT). For completeness, the procedure is briefly explained below.

All signals are divided in short blocks of data and the FFT’s of these blocks are calculated. The product of one FFT with the conjugate of another will result in a CPSD. Averaging for more blocks will reduce the variance of the resulting spectrum. To increase the length of the blocks they are half overlapping. To reduce end-effects, a smoothing window is applied.

A Moving Average (MA) approach

The FFT-blocks have been used to make the algorithm recursive in time (i.e. moving average). The idea is to include newly measured block of data and exclude the oldest block of data from the average. The calculation scheme is depicted in figure 3. After the CPSD has been refreshed, a new spreading function can be calculated.

Taking into account low Frequency Yaw Motions

Normally EMLM would be applied to a geometrically fixed array of wave buoys. Low frequent translation of the buoys would not be a problem since the correlation functions between the measured signals (which is a function of direction and frequency only) would stay the same. For faster motions Doppler effects would have to be expected.

However, the Low frequent yawing is to be considered as a problem since it changes the direction of the incoming waves with respect to the ship fixed coordinate system. This means the correlation between the probes will change over time and the EMLM will lose track.

This problem has been overcome by assuming that the yawing motion is slow enough to consider the heading of the ship constant over a short period of time. The FFT blocks as discussed before are sorted by their mean heading and then averaged for each heading.

Calculation of the mean wave drift force

For the ‘Loch Rannoch’ 3D diffraction theory calculations have been performed. From these calculations the Quadratic Transfer Function for the wave drift forces was obtained. Being a second order phenomenon, the wave drift forces and moments vary with the squared wave elevation. Multiplication of the spectrum with the main diagonal of the QTF for each direction results in the mean wave drift force:

\[
\bar{F} = 2 \sum_{i=1}^{N} \sum_{j=1}^{M} P(\omega_i, \omega_j) \cdot S(\omega_i, \theta_i) \cdot d\omega \cdot d\theta
\]

Eq 2

TEST CASES

Numerically generated data

Data has been generated using standard IFFT techniques. Test data for the yawing ship has been generated by interpolation between time traces with waves coming from different directions. An artificial heading history has been predefined for testing, see figure 2.
The ship initially has the waves at 20 degrees off the port bow (200 degrees relative wave direction), after half an hour it is changed to 45 degrees. Then a full turn is made in approximately one hour. This is thought to be adequate to test the algorithm behavior with respect to the yawing motion.

Data has been generated for a yawing array of wave buoys and for the yawing vessel. The used spectrum was a 9 sec JONSWAP spectrum with a \( \cos^{28} \) spreading function and significant waveheight of 5 m.

**Full scale measurement data**

The full-scale measurements have been performed on board of the ‘Loch Rannoch’ in a series of trials on 22 dec. 2001. For validation of the DSF the vessel was equipped with a MIROS wave radar system. This system tends to give a good estimation of the DSF. A nearby wave buoy has been used to validate the spectral energy.

**RESULTS**

**Numerically generated data**

Drift forces and moments have been plotted (figs 3, 5, and 7) against time to compare the estimates with theoretical values. The forces have been plotted in X and Y direction, but also in a cylindrical coordinate system (direction and magnitude). All results presented here are relative to the ship fixed coordinate system.

The estimated spectra have been given for fore- and aft-ship (figs 4, 6 and 8). Below these, the weighted mean of fore- and aft-ship are given. Finally the theoretical spectrum is plotted. This is the spectrum, which was used to generate the time-traces. The depicted estimated spectrum corresponds to the final time step in the force plot.

Figures 4 and 5 depict the estimate from a yawing array of wave buoys. These wave buoys have the same distance as the sensors on the vessel.

Figures 6 and 7 show the results for relative motions on the vessel. These results have been calculated using phase information only. Figures 8 and 9 show the same results as 6 and 7, only these are for the least-squares amplitude estimator.

**Full scale measurement data**

Figure 10 shows the estimated spectrum for the full-scale measurements. The estimated spectrum based on amplitude is plotted next to the MIROS Wavex spectrum from wave radar measurements. The MIROS Wavex spectrum has been scaled using the significant wave height of the wave buoy.

**DISCUSSION**

If the heading of the vessel changes, at first only the newly measured data is used to estimate a DSF for that direction. Initially, this leads to scatter in the result since one block of data has not enough observations to calculate a representative spectrum, so it may contain very high peaks. These will disturb the average DSF for all directions. To avoid this effect, estimates obtained from 3 blocks and less have been excluded from the average. The remaining scatter should be related to (1) the variability of a stochastic quantity observed in a limited time period, and (2) statistical inaccuracy in the method.

Due to the phenomenon of spatial aliasing the distance between the probes is limited to a maximum. Spatial aliasing occurs when multiple waves of one frequency fit between two measurement locations. As a result of this, the CPSD matrix is not unique for one direction anymore and the EMLM estimate will contain more directional peaks. In the current measurement configuration the distance is 40m. This means spatial aliasing will occur for a wave with a frequency of 0.9 rad/s and higher. Reducing the sensor spacing would improve the EMLM results significantly for the higher frequencies.

The implementation of the transfer functions introduced another form of aliasing. The transfer functions between the local wave and the relative motion introduce an extra phase lag between the signals. As a result of this, the CPSD is not unique over the directions anymore and aliasing occurs, see figure 7. The transfer function aliasing effect is so strong that EMLM can not produce a reliable estimate based on the phase information only.

The estimate based on the amplitude ratios between the signals produces much better results, see figures 8 and 9. However, the chosen amplitude ratios also suffer from non-uniqueness over the directions: a small hump is visible at 90 degrees. For the full-scale measurements the hump was even larger. Different combinations of probes may be taken to reduce this effect.

The total wave energy \( (M_0) \) is estimated using an indirect method. This is unavoidable since the wave height is measured only indirectly. It can be seen from the figures 4, 6 and 8 that a small error in the significant wave height \( (H_{1/3}) \) causes a large error in the force. This is due to the fact that forces depend on the squared wave height. A better estimator for \( M_0 \) (or \( H_{1/3} \)) would improve the result significantly, since the force direction in figures 4 and 8 is correct and the error is mainly caused by the error in \( H_{1/3} \).

The suppression of estimated noise coming from other than the main wave direction reduces the resolution power with respect to swells. Swells lower than a user specified energy percentage of the wind spectrum will be regarded as
noise and can not be seen. It is believed that the method based on relative wave amplitudes is not at all capable of estimating swells. This has to be further investigated.

Another option to improve the phase based EMLM results is by using different combinations of signals. Measured signals could be combined into a new signal in such a way that aliasing is avoided. As an example of this the roll motion has been subtracted from the relative motions and this indeed reduced the aliasing.

CONCLUSIONS

The EMLM has been successfully adjusted to estimate a directional spectrum with a yawing array of sensors.

Transfer functions for relative wave height have been implemented, which caused transfer function aliasing. An estimate based on relative wave height amplitude ratios produced significantly better results.

Although not yet consistent for all directions, the results look very promising for application in a wave feed forward control system.

REFERENCES


FIGURE 3: Calculation scheme for Phase- (left) and Amplitude- (right) estimator
Figure 4: Resulting driftforces from array of wave buoys

Figure 5: Resulting spectrum from wave buoys: final time step
Figure 6: Resulting spectrum from relative motions using phase information

Figure 7: Resulting spectrum from relative motions using phase information: final time step
Figure 8: Resulting drift forces result from relative motions using amplitude information.

Figure 9: Resulting spectrum from relative motions using amplitude information: final time step.
Figure 10: Forces from full scale measurements

Figure 11: Spectrum from Full scale measurements: final time step