THE LOW FREQUENCY MOTIONS OF A SEMI-SUBMERSIBLE IN WAVES

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SUMMARY

In this paper attention is paid to low frequency wave drift forces on a semi-submersible moored in irregular head waves and to the low frequency horizontal motions caused by the drift forces.

The wave drift forces are computed based on three-dimensional potential theory using the direct integration method. Time records of the low frequency drift force in irregular waves are computed using the second order impulse response function technique. Results of this method are compared with results of the direct summation method. The comparison shows that use of the second order impulse response function leads to numerically accurate results. A computed low frequency drift force record is compared with a measured drift force record. The comparison shows that the predictions are qualitatively correct but some 30 to 40 per cent lower than measured data. Results of time domain simulation computations of the low frequency surge motions in irregular head seas are compared with measurements. A discussion is given regarding differences between computations and measurements.
NOMENCLATURE

\[
\begin{align*}
\alpha_{11} &= \text{added mass in surge} \\
\beta_{11} &= \text{linear damping coefficient in surge} \\
\beta_2 &= \text{quadratic damping coefficient} \\
c &= \text{restoring coefficient of the mooring system} \\
dl &= \text{length element of the waterline} \\
dS &= \text{surface element of } S \text{ or } S_0 \\
F &= \text{force vector with components } F_1, F_2 \text{ and } F_3 \\
F^{(2)} &= \text{second order force vector with components } F^{(2)}_1, F^{(2)}_2 \text{ and } F^{(2)}_3 \\
g^{(2)}(t_1, t_2) &= \text{quadratic impulse response function} \\
M &= \text{mass matrix} \\
m &= \text{mass of the vessel in air} \\
\bar{n} &= \text{outward pointing normal unit vector of a surface element } dS \text{ relative to a right-handed system of co-ordinates with origin in the centre of gravity and } x_3\text{-axis, vertically upwards} \\
\bar{n} &= \text{outward pointing normal unit vector of a surface element } dS \text{ referenced to the body axes with components } n_1, n_2 \text{ and } n_3 \\
n_1(t) &= \text{direction cosine of a length element } dl \text{ in longitudinal direction} \\
P_{ij} &= \text{in-phase part of the quadratic transfer function} \\
P &= \text{pressure obtainable from Bernoulli's equation} \\
Q_{ij} &= \text{quadrature part of the quadratic transfer function} \\
S &= \text{instantaneous wetted surface} \\
S_0 &= \text{mean wetted surface} \\
T_{ij} &= \text{amplitude of the quadratic transfer function} \\
t &= \text{time} \\
t_1, t_2 &= \text{time shifts} \\
X^{(1)} &= \text{first order motion of a surface element } dS \\
X^g &= \text{motion vector of the centre of gravity with components } x^g_1, x^g_2 \text{ and } x^g_3 \\
\bar{X}^{(1)} &= \text{first order oscillatory component of the motions of the centre of gravity} \\
\bar{a}^{(1)} &= \text{first order angular motion vector with components } x^{(1)}_4 (\text{roll}), x^{(1)}_5 (\text{pitch}) \text{ and } x^{(1)}_6 (\text{yaw}) \\
\xi_\epsilon &= \text{random phase uniformly distributed over } 0 - 2\pi \\
\epsilon^{(1)}(t) &= \text{phase angle of the relative wave elevation at point } i \text{ related to the undisturbed wave crest passing the centre of gravity} \\
\zeta^{(1)} &= \text{amplitude of the } i\text{-th regular wave component} \\
\zeta^{(1)}(t) &= \text{time dependent wave elevation} \\
z^{(1)}(t) &= \text{transfer function of the amplitude of the first order relative wave elevation at point } i \text{ in the waterline} \\
z^{(1)}(t, \xi_i) &= \text{time dependent relative wave elevation in a point } i \text{ along the waterline}
\end{align*}
\]
Morison's equation are of significant importance and that the drift forces are proportional to period. Results of calculation indicate that viscous effects arising from the drag term in the construction is essentially left unsettled. Head waves will be presented. The time domain record of the measured wave drift force in irregular waves is compared with results of computation. The computations are based on three-dimensional potential theory and take into account the hydrodynamic interaction between the elements. These forces, also known as low frequency wave drift forces, are associated with the capabilities of a structure to reflect waves and can generate large amplitude low frequency motions. In none of these cases are results of computation influenced and the corresponding motion could be optimised, for instance, by alteration in thruster sizing.

The use of larger diameter columns and the increase in displacement, however, also result in a potentially greater capability of the structure to disturb and reflect the incoming waves. This may have a significant effect on the low frequency second order wave forces on the structure. These forces, also known as low frequency wave drift forces, are associated with the capabilities of a structure to reflect waves and can generate large amplitude low frequency motions in moored vessels resulting in high peak mooring loads. In the case of dynamically positioned vessels, these forces are of importance from the point of view of station keeping accuracy and thruster sizing.

In order to investigate the merits of a particular design for a semi-submersible from the point of view of wave drift forces and the resultant motion, a theory which embodies all relevant aspects and which can be used as a basis for computation, is needed. In the past, a number of theories have been put forward by means of which the wave drift forces on a semi-submersible could be computed. See for instance Wahab [2], Pijfers and Brink [3], Ferretti and Berta [4] and Karppinen [5]. The last of these authors presented a computational method which is based on potential theory and the assumption that the elements of a semi-submersible such as the columns and floaters are slender and from a hydrodynamic point of view, do not interact. The total second order wave drift force is then the sum of the drift forces on the elements in the absence of all other elements. According to potential theory which disregards viscous effect, the drift force is a quadratic function of the wave height. Wahab [2], Pijfers and Brink [3] and Ferretti and Berta [4] also make use of the assumption of slenderness and the absence of hydrodynamic interaction of the elements of the semi-submersible. The hydrodynamic force in each element is determined through the use of Morison's equation and the relative velocity between the fluid and the elements. The total force is found by summation over the elements. The drift force is defined as the mean value of the total force averaged over a wave period. Results of calculation indicate that viscous effects arising from the drag term in Morison's equation are of significant importance and that the drift forces are proportional to about the third power of the wave height. In none of these cases are results of computation compared with experimental results, however, so the issue as to whether the wave drift forces on semi-submersible structures are significantly influenced by viscous effects or that these forces may be determined by methods which neglect hydrodynamic interaction between the elements of the construction is essentially left unsettled.

In this paper experimental results of model tests with a semi-submersible in irregular head waves will be presented. The time domain record of the measured wave drift force in irregular waves is compared with results of computation. The computations are based on three-dimensional potential theory and take into account the hydrodynamic interaction between the elements.
Results of time domain simulation of the low frequency surge motions in irregular head waves are also compared with experimental results. Before comparing the results of computations and experiments a brief account will be given of the method of computation employed for determination of the wave drift forces and the simulation techniques used to determine the low frequency components of the wave drift force and the horizontal surge motion of the semi-submersible.

**WAVE DRIFT FORCES**

Computations of wave drift forces are based on the direct integration method, see ref. [6] and [7]. By this method the wave drift forces are found from the second order term in the following expression for the hydrodynamic force:

\[ F = - \int p \cdot n \, ds \]  

(1)

Following the development given in ref. [6] we obtain the following expression for the second order wave force:

\[ F^{(2)} = \int_{WL} \hat{p} \hat{g} (1)^2 \cdot n \, ds + \hat{a} (1) \times (M \cdot \hat{X}_g) + \int_0 \left\{ -\hat{p} \left[ \hat{V}_t (1) \right]^2 - \rho \left( \phi^{(2)}_w + \phi^{(2)}_d \right) \right\} \cdot n \, ds \]  

(2)

For the purpose of time domain simulation of wave drift forces equation (2) is unsuitable in its present form. This is due to the large amount of data which must be computed at each time step in order to be able to carry out the integrations around the waterline and over the hull surface. It is computationally more convenient to use equation (2) to compute frequency domain quadratic frequency response or transfer functions, which in turn can be transformed into time domain second order impulse response functions. These second order impulse response functions, when convoluted with the undisturbed wave train, yield time records of the second order drift forces. See ref. [8]. This procedure makes it possible to compute wave drift force records for arbitrary wave records in a relatively straightforward and economic manner.

In the following a brief description is given of the procedure to obtain the quadratic transfer function for the second order wave drift forces.

**QUADRATIC TRANSFER FUNCTION**

The total quadratic transfer function is split up in contributions arising from the following components of equation (2):

1: First order relative wave elevation
   \[ - \hat{p} \hat{g} \int_{WL} \hat{z} (1)^2 \cdot n \, ds \]  
   (3)

2: Pressure drop due to first order velocity
   \[ \int_0 \left\{ -\hat{p} \left[ \hat{V}_t (1) \right]^2 - \rho \left( \phi^{(2)}_w + \phi^{(2)}_d \right) \right\} \cdot n \, ds \]  
   (4)

3: Pressure due to product of gradient of first order pressure and first order motion
   \[ \int_0 -\rho \left( \hat{X} (1) \cdot \hat{V}_t (1) \right) \cdot n \, ds \]  
   (5)

4: Contribution due to products of first order angular motions and inertia forces
   \[ \hat{a} (1) \times (M \cdot \hat{X}_g) \]  
   (6)

5: Contribution due to second order potentials
   \[ \int_0 -\rho \left( \phi^{(2)}_w + \phi^{(2)}_d \right) \cdot n \, ds \]  
   (7)

The procedure to obtain the quadratic transfer functions of the forces dependent on first order quantities (I, II, III and IV) will be illustrated by taking the low frequency part of the longitudinal component of the force contribution due to the relative wave elevation:
In irregular long-crested waves the elevation, to first order, of the incoming undisturbed waves - referred to the mean position of the centre of gravity of the floating body - may be written as:

$$\zeta^{(1)}(t) = \sum_{i=1}^{N} \zeta_i^{(1)} \cos(\omega_i t + \xi_i)$$

(9)

The first order relative wave elevation at a point on the waterline of the body may be written as follows:

$$\zeta_i^{(1)}(t, x) = \sum_{i=1}^{N} \zeta_i^{(1)} \cdot \zeta_i^{(1)'}(x) \cdot \cos(\omega_i t + \xi_i + \epsilon_i)$$

(10)

Substitution of (10) in equation (8) leads to:

$$\begin{align*}
F_{1_{ij}}^{(2)}(t) &= \sum_{i=1}^{N} \sum_{j=1}^{N} \zeta_i^{(1)} \cdot \zeta_j^{(1)} \cdot P_{ij} \cdot \cos((\omega_i - \omega_j) t + (\xi_i - \xi_j)) + \\
&\quad + \sum_{i=1}^{N} \sum_{j=1}^{N} \zeta_i^{(1)} \cdot \zeta_j^{(1)} \cdot Q_{ij} \cdot \sin((\omega_i - \omega_j) t + (\xi_i - \xi_j)) + \\
&\quad + \text{high frequency terms}
\end{align*}$$

(11)

where $P_{ij}$ and $Q_{ij}$ are the in-phase and out-of-phase components of the time independent transfer function with:

$$\begin{align*}
P_{ij} &= P_i(\omega_i, \omega_j) = \int_{-L}^{L} \log \zeta_i^{(1)}(x) \cdot \zeta_j^{(1)}(x) \cos(\xi_j(x) - \xi_i(x)) n(x) \, dx \\
Q_{ij} &= Q_i(\omega_i, \omega_j) = \int_{-L}^{L} \log \zeta_i^{(1)}(x) \cdot \zeta_j^{(1)}(x) \sin(\xi_j(x) - \xi_i(x)) n(x) \, dx
\end{align*}$$

(12)

(13)

Taking the low frequency part of the square of the wave elevation given by equation (9) results in:

$$\zeta^{(1)}(t, x)^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} 4 \zeta_i^{(1)} \cdot \zeta_j^{(1)} \cdot \cos((\omega_i - \omega_j) t + (\xi_i - \xi_j))$$

(14)

Comparison with equation (11) shows that $P_{ij}$ and $Q_{ij}$ are transfer functions which give that part of the wave drifting force which is in-phase and out-of-phase respectively with the low frequency part of the square of the incident wave elevation.

It will be clear that similar developments can be made for other contributions to the wave drifting forces which depend only on first order quantities. The contribution $V$ due to second order potentials is approximated using results of first order wave loads; see ref. [7]. The total in-phase and out-of-phase transfer functions are found by simple summation of the contributions from the five components. The wave drifting forces may thus be presented as transfer functions which, as can be seen from the foregoing, are a function of two frequencies. In general, the quadratic transfer functions will also be functions of the direction of the waves.

Based on a wave elevation as given by equation (9) the total wave drift force is found from:

$$\begin{align*}
P_{1_{ij}}^{(2)}(t) &= \sum_{i=1}^{N} \sum_{j=1}^{N} \zeta_i^{(1)} \cdot \zeta_j^{(1)} \cdot P_{ij} \cdot \cos((\omega_i - \omega_j) t + (\xi_i - \xi_j)) + \\
&\quad + \sum_{i=1}^{N} \sum_{j=1}^{N} \zeta_i^{(1)} \cdot \zeta_j^{(1)} \cdot Q_{ij} \cdot \sin((\omega_i - \omega_j) t + (\xi_i - \xi_j))
\end{align*}$$

(15)

in which $P_{ij}$ and $Q_{ij}$ are found by summation of contributions I through V.
DRIFT FORCE IN A REGULAR WAVE GROUP

We consider the first order wave elevation in a regular wave group consisting of two regular waves with frequency $\omega_1$ and $\omega_j$:

$$\zeta^{(1)}(t) = \sum_{i=1}^{2} \zeta_i^{(1)} \cos(\omega_i t + \epsilon_i)$$

$$= \zeta_1^{(1)} \cos(\omega_1 t + \epsilon_1) + \zeta_2^{(1)} \cos(\omega_2 t + \epsilon_2)$$

(16)

The second order force associated with such a wave train has the following form:

$$F_1^{(2)}(t) = \sum_{i=1}^{2} \sum_{j=1}^{2} \zeta_i^{(1)} \cdot \zeta_j^{(1)} \cdot P_{ij} \cdot \cos((\omega_i - \omega_j)t + (\epsilon_i - \epsilon_j))$$

$$+ \sum_{i=1}^{2} \sum_{j=1}^{2} \zeta_i^{(1)} \cdot \zeta_j^{(1)} \cdot Q_{ij} \cdot \sin((\omega_i - \omega_j)t + (\epsilon_i - \epsilon_j))$$

$$= \zeta_1^{(1)} \cdot P_{11} + \zeta_2^{(1)} \cdot P_{22}$$

$$+ \zeta_1^{(1)} \cdot \zeta_2^{(1)} \cdot (P_{12} + P_{21}) \cdot \cos((\omega_1 - \omega_2)t + (\epsilon_1 - \epsilon_2))$$

$$+ \zeta_1^{(1)} \cdot \zeta_2^{(1)} \cdot (Q_{12} - Q_{21}) \cdot \sin((\omega_1 - \omega_2)t + (\epsilon_1 - \epsilon_2))$$

(17)

From equation (17) it is seen that the second order force contains two constant components. Each of these components represents the constant force which would be found if the wave train consisted of a single regular wave with frequency $\omega_1$ or $\omega_2$ respectively. This shows that, although the force is a non-linear phenomenon, the constant or mean second order force in a wave train consisting of a superposition of regular waves is the sum of the mean forces found for each of the component waves. The quadratic transfer function:

$$P(\omega_1, \omega_2)$$

gives the mean second order force in regular waves with frequency $\omega_1$. In literature dealing with the mean second order forces on floating objects in regular or irregular waves this is often expressed as a function dependent on one frequency $\omega_1$. The above equations show that the transfer function for the mean or constant part is, however, only a specific case of the general quadratic transfer function $P(\omega_1, \omega_2)$ for the force in regular wave groups.

Besides the constant parts the second order force contains low frequency parts with a frequency corresponding to the difference frequency $\omega_1 - \omega_2$ of the component regular waves. It is seen that the amplitudes of the in-phase and out-of-phase parts depend on the sum of the in-phase quadratic transfer functions $P_{12}$ and $P_{21}$ and the difference of the out-of-phase functions $Q_{12}$ and $Q_{21}$.

SYMMETRY OF THE QUADRATIC TRANSFER FUNCTIONS

From equation (17) it is seen that the transfer functions do not appear in isolation but rather in pairs. In general, the in-phase and out-of-phase components of the quadratic transfer functions as determined from equations (3) through (7) for combinations of $\omega_1$ and $\omega_2$ will be so that, for instance:

$$P(\omega_1, \omega_2) \neq P(\omega_2, \omega_1)$$

(19)

However, since the force as given in equation (17) depends on the sum or difference of the components of the quadratic transfer functions these may be so reformulated that the following symmetry relations are valid:

$$P(\omega_1, \omega_2) = P(\omega_2, \omega_1)$$

(20)

$$Q(\omega_1, \omega_2) = -Q(\omega_2, \omega_1)$$

(21)

The in-phase component $P(\omega_1, \omega_2)$ of the quadratic transfer function of the total second order
force takes the form of a matrix which is symmetrical about the diagonal for which \( w_1 \) is equal to \( w_2 \) while the out-of-phase component \( Q(w_1,w_2) \) is anti-symmetrical about the diagonal.

**EVALUATION OF QUADRATIC TRANSFER FUNCTIONS**

Evaluation of the various components of the quadratic transfer functions of the low frequency wave drifting forces requires detailed knowledge of the first order vessel motions and fluid motions. For instance, as shown by equations (12) and (13), evaluation of contribution 1 requires knowledge of the relative wave elevation amplitudes and phase angles around the waterline.

A numerical method by means of which such detailed information may be obtained (using a distribution of sources over the mean wetted surface of the body) has been developed by Boreel [9] and Van Oortmerssen [10].

**TIME DOMAIN REPRESENTATION OF THE MEAN AND LOW FREQUENCY SECOND ORDER FORCES**

According to Dalzell [8] the low frequency second order forces can be computed given the quadratic transfer function and the time record of the wave elevation using the following relationship:

\[
F_1^{(2)}(t) = \int_0^\infty \int_0^\infty g^{(2)}(t_1,t_2) \cdot \xi^{(1)}(t-t_1) \cdot \xi^{(1)}(t-t_2) \cdot dt_1 \cdot dt_2
\]  

(22)

The quadratic impulse response function \( g^{(2)}(t_1,t_2) \) is derived from the following expression:

\[
g^{(2)}(t_1,t_2) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i(w_1 t_1 + i w_2 t_2)} \cdot G^{(2)}(w_1,w_2) \cdot dw_1 \cdot dw_2
\]  

(23)

in which:

\[
G^{(2)}(w_1,w_2) = \text{complex quadratic transfer function}
\]

\[
P_{w_1,w_2} + iQ_{w_1,w_2}
\]  

(24)

From equation (22) it is seen that if the quadratic impulse response function \( g^{(2)}(t_1,t_2) \) is known the time record of the low frequency second order forces can be computed for arbitrary wave elevation records. The applicability of this technique has been demonstrated extensively and convincingly by Dalzell [11] using quadratic transfer functions for the second order forces obtained from tests in irregular waves using cross-bi-spectral analysis techniques.

Time records of second order wave drift forces in irregular waves can also be generated based on equation (15). In such cases it is assumed that the irregular waves are described by the spectral density \( S_c(w) \). The amplitudes \( \xi_k \) are found from the following relationship:

\[
\xi_k = \sqrt{2 S_c(w_k) \Delta w}
\]  

(25)

The phase angles \( \xi_k \) are random with a homogeneous distribution function from zero to 2\( \pi \). The wave elevation follows from equation (9). The wave drift force record is found from equation (15). It should be noted, however, that if time records of drift forces are generated for the purpose of simulating the behaviour of moored vessels, the frequency differences \( w_1 - w_2 \) in equation (15) must be chosen sufficiently small in order to cover the frequency range of significant response of the moored vessel with enough frequency bands. This will generally result in a large number, \( N \), of discrete frequencies \( w_k \) being used to describe the wave train of equation (9). This in turn leads to a large number of summations to be carried out when generating wave drift forces according to equation (15).

Computational efforts involved with evaluation of the wave drift force record according to equation (15) can be drastically reduced by choosing equidistant frequencies \( w_k \) to describe the wave spectrum. If the frequency step used to describe the wave spectrum is \( \Delta w \) then the wave record and hence the wave drift force record become periodic with a period of:

\[
T = \frac{2\pi}{\Delta w}
\]  

(26)

According to Fourier's theorem the wave drift force record can then be written in the form of a single summation as follows:

\[
F_1^{(2)}(t) = \sum_{k=0}^{M} \{ A_k \cos w_k t + B_k \sin w_k t \}
\]  

(27)
where: \( \omega_k = k \cdot \omega \) 

and from equation (15) \( A_k \) and \( B_k \) are found to be:

\[
A_k = \delta_k \sum_{j=1}^{N-k} \zeta_j \cdot \zeta_j \{ P_{j+k,j} \cdot \cos(\xi_{j+k} - \xi_j) + Q_{j+k,j} \cdot \sin(\xi_{j+k} - \xi_j) \}
\]

\[
B_k = \delta_k \sum_{j=1}^{N-k} \zeta_j \cdot \zeta_j \{ Q_{j+k,j} \cdot \cos(\xi_{j+k} - \xi_j) - P_{j+k,j} \cdot \sin(\xi_{j+k} - \xi_j) \}
\]

with:

- \( \zeta_k = 1 \) for \( k = 0 \)
- \( \zeta_k = 2 \) for \( k \neq 0 \)

\( N = \) number of wave frequencies used to describe the wave spectrum 
\( M = N - 1 \)

Use of equation (27) instead of equation (15) involves computing the coefficients \( A_k \) and \( B_k \) according to equations (29), (30) and (25) using data on the quadratic transfer functions \( P_{ij} \) and \( Q_{ij} \). Once these coefficients are computed and stored the time record of the wave drift force is computed from the single summation of equation (27).

In this paper time records of wave drift forces on a semi-submersible in irregular head waves are given, which have been computed using both the quadratic impulse response function technique and the direct summation technique of equation (27).

COMPUTATIONS OF LOW FREQUENCY MOTIONS

In this paper the low frequency wave drift force induced surge motions of a semi-submersible in head seas will be presented. The equation of motion describing the surge motion is as follows:

\[
(m + a_{11}) \ddot{x}_1 + b_{11} \dot{x}_1 + b_2 \dot{x}_1 \cdot | \dot{x}_1 | + c x_1 = F^{(2)}_1(t)
\]

The added mass \( a_{11} \) was determined based on three-dimensional potential theory computations. The damping coefficients \( b_{11} \) and \( b_2 \) were obtained from still water surge motion decay tests in the mooring system. The time record of the wave drift force \( F^{(2)}_1(t) \) is computed off-line and stored. After this the equation of motion is solved for the surge motion.

The aforementioned equation of motion is rather simple. For more complex cases involving more degrees of freedom, and in which the assumption of constant added mass and damping is not valid anymore, a more suitable set of equations of motion is chosen. See Van Oortmerssen [10].

THE VESSEL

Computations and model tests have been carried out for a six column, two floater semi-submersible. The main particulars are given in Table I and Figure 1. The model was made of PVC to a scale of 1 to 40.

For the computations using three-dimensional diffraction theory, the wetted surface of the hull is approximated using a total of 216 plane facet elements. The waterline of the semi-submersible is approximated using a total of 72 straight line elements. The facet schematisation of one floater and the waterline schematisation of one column are shown in Figure 2.

Model tests and computations were carried out for a water depth corresponding to 40 m full scale.

MODEL TESTS

Model tests (and computations) have been carried out for this vessel in a variety of conditions. In ref. [7] results have been given on tests in regular waves from ahead, abeam and from the bow quartering direction. In the reference mentioned, mean drift forces in regular waves and wave frequency motion response functions have been compared with results of computations. The comparisons confirm that, in general, the wave frequency motions and mean drift forces are accurately predicted by the computations. For further details we refer to the above mentioned reference.

For this paper attention is restricted to tests in irregular head seas. Two tests will be discussed here, viz.:

- One test to measure the time record of the surge wave drift force (wave drift force test).
- One test to measure the low frequency surge motions with the vessel moored in a soft linear mooring system (motion test).

Besides tests in irregular head seas a surge motion decay test was carried out in still water with the vessel moored in the soft linear mooring system. In Figure 3 the wave spectrum is shown of the irregular waves for which model tests were carried out.

**TEST SET-UP FOR THE WAVE DRIFT FORCE TEST**

The purpose of this test was to measure the wave drift forces in irregular waves in such a way that the results would be representative for the wave drift forces acting on a soft moored semi-submersible. This required a test set-up which, on the one hand does not restrict the wave frequency motions, and on the other hand restricts as much as possible low frequency surge motions. The latter requirement ensures that the measured forces are not affected unduly by dynamic magnification effects.

In order to measure the low frequency wave drift force on the semi-submersible use was made of a stiff linear spring mooring system. The set-up is shown schematically in Figure 4. The stiffness of the springs was chosen such that the natural frequency of the surge motion was just outside the frequency range of the irregular waves. The natural surge frequency corresponded with 0.4 rad./sec. full scale. In spite of the relatively large stiffness of the mooring system, the wave frequency motions were only slightly affected in the range of the wave frequencies present in the spectrum. This is shown in Figure 5 in which results of wave frequency heave, pitch and surge motion response computations are shown with and without the effect of the stiff mooring system. In Figure 6 the effect of the mooring stiffness on the computed values of the mean drift force in regular waves is shown. Again the influence of the mooring stiffness is small, which indicates that the low frequency wave drift force on the vessel in the stiff mooring system is practically the same as it would be on the vessel moored in a soft system.

The mean and low frequency wave drift force was found from the mooring force measurements (see Figure 4). Due to the mooring system the measured low frequency force will tend to suffer from magnification effects as the frequency of interest increases from zero upward to the natural surge frequency. For the present case, low frequency drift forces with frequencies up to 0.1 rad./sec. full scale suffered less than 6 per cent from dynamic magnification effects. This means that the low frequency components of the measured mooring force could be equated to the wave drift force up to a frequency of 0.1 rad./sec. full scale. Above this frequency magnification effects increased above 6 per cent.

**TEST SET-UP FOR THE MOTION TEST**

The principle of the test set-up for this test is also shown in Figure 4. In this case, however, the mooring stiffness was considerably less. The natural frequency of the surge motion amounted to 0.036 rad./sec. full scale, which corresponds to a natural period of 176 seconds. Surge motion decay tests to determine the still water damping coefficients \( b_1 \) and \( b_2 \) of equation (31) were also carried out in the soft spring system. From these tests the following values were found:

\[
\begin{align*}
  b_{11} & = 8 \text{ tf.sec./m} \\
  b_2 & = 100 \text{ tf.sec}^2/\text{m}^2
\end{align*}
\]

**RESULTS OF COMPUTATIONS OF THE QUADRATIC TRANSFER FUNCTION AND THE SECOND ORDER IMPULSE RESPONSE FUNCTION**

In Figure 7 and Figure 8 plots are given of the quadratic transfer functions \( P_{ij} \) and \( Q_{ij} \) respectively of the drift force in head waves. The plots show the contour lines of the functions \( P_{ij} \) and \( Q_{ij} \) in the \( \omega_i, \omega_j \) frequency plane. In each plot numbers have been assigned to the contours. These express the values of \( P_{ij} \) and \( Q_{ij} \) in metric tons/m². The plots only cover part of the total \( \omega_i, \omega_j \) plane. For \( |\omega_i - \omega_j| > 0.24 \) the transfer functions have not been computed and have been set to zero. The rapid fall-off of the functions along the lines \( |\omega_i - \omega_j| = 0.24 \) is therefore not a characteristic of the functions but due to truncation. Data for larger differences frequencies are not relevant for the low frequency surge motions. For other modes of motion, for instance, when low frequency vertical motions are important, it may be necessary to enlarge the transfer functions to larger difference frequencies. The function \( P_{11} \) given in Figure 7 is symmetrical about the line \( \omega_i = \omega_j \) while the function \( Q_{11} \) given in Figure 8 is antisymmetrical about the line \( \omega_i = \omega_j \). This is in keeping with equations (20) and (21). In Figure 7 the data on the line \( \omega_i = \omega_j \) corresponds with the mean drift forces in regular waves shown in Figure 6. The negative sign of \( P_{ij} \) indicates that the mean force in head waves is directed aft. It is noted that the function \( P_{11} \) in Figure 7 generally has its lowest values on the diagonal \( \omega_i = \omega_j \). This function tends to increase as the difference frequency \( |\omega_i - \omega_j| \) is increased. Below a frequency of about 0.75 rad./sec. there are no contour lines except near the frequency...
0.3 rad./sec. At this frequency a slight "dip" is shown in $P_{14}$. This frequency corresponds to the natural heave and pitch frequency of the vessel. In between 0.75 rad./sec. and 0.3 rad./sec., the function $P_{14}$ remains above the value of -2 and below zero. The first contour line marks the -2 level, hence no contour lines of $P_{14}$ are shown in this interval. The decrease (larger negative values indicating that the aft directed force is increasing) in the value of $P_{14}$ above frequencies of about 0.75 rad./sec. is associated with increasing diffraction effects. The structure becomes large relative to the wave length and more wave energy is reflected back. The peaks and troughs shown in Figure 7 are due to changing interaction effects occurring between the six columns of the semi-submersible. This is also reflected in Figure 6, in which the mean drift force in regular waves is shown. The results in this figure correspond to the values on the diagonal of Figure 7.

As indicated previously, the function $Q_{14}$ shown in Figure 8 is anti-symmetric about the diagonal. On the diagonal the values are zero. As one moves further away from the diagonal the values, in absolute sense, increase. For low values of $\omega_1$ and $\omega_4$, $Q_{14}$ increases more rapidly as $|\omega_1 - \omega_4|$ increases. This is due to contribution $V$ to the drift force which is given in equation (7). For higher values of $\omega_1$ and $\omega_4$, $Q_{14}$ shows peaks and troughs which are again associated with diffraction and interaction effects between columns of the semi-submersible. For $|\omega_1 - \omega_4| > 0.24$, $Q_{14}$ falls off rapidly. This is again due to truncation.

In Figure 9 the amplitude $T_{ij}$ of the wave drift force transfer function is given. This is defined by:

$$T_{ij} = \left(P_{ij}^2 + Q_{ij}^2\right)^{1/2}$$

(32)

By definition $T_{ij}$ is symmetric about the diagonal. Inspection of $T_{14}$ in comparison with $P_{14}$ and $Q_{14}$ shows that for higher values of $\omega_1$ and $\omega_4$ the amplitude $T_{14}$ is dominated by the in-phase part $P_{14}$. For lower values of $\omega_1$ and $\omega_4$, the quadrature part $Q_{14}$ is dominant. For frequency values greater than about 0.75 rad./sec., $P_{14}$ values decrease rapidly. Between 0.75 rad./sec. and 0.95 rad./sec. the contours of $T_{14}$ tend to run at right-angles to the diagonal $\omega_1 = \omega_4$ for $|\omega_1 - \omega_4|$ values of up to 0.1 rad./sec. This type of behaviour tends to favour the use of approximate methods for computing drift forces in irregular waves as, for instance, given by Pinkster [12]. This is because in such cases the off-diagonal values of $T_{ij}$ can be replaced by the value on the diagonal $T_{nn}$ where the frequency $\omega_n$ is equal to the mean of the frequencies $\omega_i$ and $\omega_j$. For lower values of the frequencies $\omega_i$ and $\omega_j$ the values of $T_{ij}$ are changing more rapidly when moving away from the diagonal $\omega_i = \omega_j$. This tends to make approximative methods, which are based on the data on the diagonal only, less acceptable.

Figures 7, 8 and 9 show that the quadratic transfer function is a complicated surface in the $\omega_i, \omega_j$ plane. Some of its features are readily associated with known physical effects. A full understanding of the quadratic transfer function will require more systematic computations.

In Figure 10, the second order impulse response function $g^{(2)}(t_1, t_2)$ computed according to equation (23) is shown. The function $g^{(2)}(t_1, t_2)$ is shown for $t_1$ and $t_2$ values from -30 sec. to +30 sec. This is only part of the complete function which was computed for $t_1$ and $t_2$ from -78 sec. to +78 sec. with a sample time of 1.14 sec. The function $g^{(2)}(t_1, t_2)$ is real and symmetrical about the diagonal $t_1 = t_2$. It reaches its largest values, in absolute sense, on the diagonal. The largest value on the diagonal is -0.48 tf.m²/sec for $t_1 = t_2 = 6.8$ sec. The contours of the function generally run parallel to the diagonal indicating a strong dependence on the time difference $t_1 - t_2$. As we move away from the diagonal, $g^{(2)}(t_1, t_2)$ oscillates in sign and decays. The same behaviour was also found by Dalzell [11] when studying the quadratic impulse response function for the added resistance of a vessel travelling in head waves.

The function $g^{(2)}(t_1, t_2)$ is too complicated to be able to identify any particular physical phenomena as having influence on a particular feature of the function. Again, systematic computations in which, for instance, the geometry of a platform is changed, can be used to get a better feel for the significance of such data.

RESULTS OF TIME DOMAIN COMPUTATIONS OF THE DRIFT FORCE

In order to verify that the wave drift force in irregular waves is computed correctly using the quadratic transfer function and equation (22), the results obtained in this way are compared with results obtained by application of the direct summation technique given by equation (27). To this end a wave train corresponding to the wave spectrum in Figure 3 was generated based on equation (9) using 600 equidistant frequency components. For this wave train the "true" wave drift force signal is obtained from equation (27) using data on $P_{14}$ and $Q_{14}$ shown in Figures 7 and 8. The wave drift force signal was also computed using the second order impulse response function $g^{(2)}(t_1, t_2)$ and equation (22). The results are compared in Figure 11. It can be seen that the agreement between the two methods is excellent, thus giving confidence in the applicability of equation (22) to arbitrary measured wave records.
COMPARISON OF MEASURED AND COMPUTED DRIFT FORCES

In Figures 12, 13 and 14 time records of the measured and computed low frequency surge drift force in irregular waves are given. In these figures the record of the waves and the low frequency part of the square of the wave elevation are also given. All records, except for the wave record, have been low-pass filtered so as to eliminate frequencies higher than about 0.1 rad./sec. full scale. The measured force record still contains some higher frequency components. This is due to the stiff mooring system used for this test, which resulted in large wave frequency force components. As was shown previously, those did not influence the wave frequency motions significantly. The record of the low-pass filtered square of the wave elevation is included to show the clear relationship between the force and this quantity. The computed record is based upon the measured wave record and equation (22).

In Figure 15 the spectral density of the measured and computed wave drift force records are shown up to a frequency of 0.1 rad./sec. full scale. The spectral densities were computed based on a record length of 6 hours full scale using the auto-correlation function method. The sample period was 3.79 seconds and 200 lags were used to compute the auto-correlation functions. No smoothing was applied.

From the comparison it can be concluded that the computed wave drift force record is in good qualitative agreement with the measured force record. The spectral densities of both records given in Figure 15 are very similar in form only, the magnitude is somewhat different. In general it appears that the computations underestimate the wave drift force both in the mean value and in the amplitude of the slowly varying components by 30 to 40 per cent. The good agreement between the direct summation technique and the second order impulse response technique (see Figure 11) indicates that in this case, the cause of the differences may lie in inaccuracies in the force measurements, inaccuracies in the computation of the quadratic transfer function or be caused by physical effects not taken into account in the computations such as forces of viscous origin. This will be a subject for further research and no definite conclusions can be drawn at this time.

COMPARISON OF COMPUTED AND MEASURED LOW FREQUENCY SURGE MOTIONS

In Figure 16 results are given of the computed and measured low frequency surge motions of the semi-submersible. The computed surge motions were determined based on equation (31) using the computed wave drift force signal. Also shown in the figure is the measured wave elevation. The duration of the motion test corresponded to 30 minutes full scale. The transient components in the computed surge take about 10 minutes to die out, thus leaving about 20 minutes for the comparison. The comparison shows that the computed surge record corresponds quite well with the measured record with respect to the phase of the motion and the general features of the record. The amplitudes of the computed record are, however, somewhat lower than the measured values. This is in agreement with the previously found differences between the measured and computed low frequency drift forces.

CONCLUSIONS

In this paper a general method to determine the low frequency wave drift forces on a vessel floating in irregular waves has been described. Application of this method to the case of a six column semi-submersible and comparison of results of time domain computations of wave drift forces and low frequency motions with results of model tests has shown that, in a qualitative sense, both the wave drift forces and the low frequency motions are well predicted by computations. The computed drift forces, however, fall below the measured drift forces by some 30 to 40 per cent. Using the computed drift forces, time domain simulations of the low frequency horizontal motions show the same order of differences compared to results of measurements.

More investigations will be carried out to determine the phenomena which may effect the differences. The computations of wave drift forces have been based on potential theory. Results given in this paper indicate that, although differences exist between results of measurements and computations, for this case at least, the low frequency wave drift forces acting on the semi-submersible are dominated by potential effects.

REFERENCES

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5. Karppinen, T., 1979, An approach to computing the second order steady forces on semi-submersible structures, Helsinki University of Technology, Ship Hydrodynamics Laboratory, Report No. 16.


TABLE I
MAIN PARTICULARS OF SEMI-SUBMERSIBLE

<table>
<thead>
<tr>
<th>Designation</th>
<th>Symbol</th>
<th>Unit</th>
<th>Value</th>
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<tbody>
<tr>
<td>Length between perpendiculars</td>
<td>Lpp</td>
<td>m</td>
<td>100.00</td>
</tr>
<tr>
<td>Breadth</td>
<td>B</td>
<td>m</td>
<td>76.00</td>
</tr>
<tr>
<td>Draught</td>
<td>T</td>
<td>m</td>
<td>20.00</td>
</tr>
<tr>
<td>Displacement volume</td>
<td>V</td>
<td>m³</td>
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</tr>
<tr>
<td>Water depth</td>
<td>Wd</td>
<td>m</td>
<td>40.00</td>
</tr>
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Fig. 1. Layout of the semi-submersible

Fig. 2. Facet schematisation

Fig. 3. The wave spectrum

Fig. 4. Mooring system
Fig. 5. Influence of mooring system stiffness on first order motions

Fig. 6. Influence of mooring system stiffness on mean drift forces
Fig. 7. In-phase component of the quadratic transfer function for the drift force in head seas

Fig. 8. Quadrature component of the quadratic transfer function for the drift force in head seas

Fig. 9. Amplitude of the quadratic transfer function for the drift force in head seas

Fig. 10. Second order impulse response function $g(t_1, t_2)$ for the drift force in head seas
Fig. 11. Comparison of computed drift forces
Fig. 12. Comparison of computed and measured drift forces
Fig. 13. Comparison of computed and measured drift forces
Fig. 14. Comparison of computed and measured drift forces
Fig. 15. Spectra of wave drift forces

Fig. 16. Low frequency motions in irregular head seas