INTRODUCTION

Stationary vessels floating or submerged in irregular waves are subjected to large, first order, wave forces and moments which are linearly proportional to the wave height and contain the same frequencies as the waves. They are also subjected to small, second order, mean and low frequency wave forces and moments which are proportional to the square of the wave height. The frequencies of the second order low frequency components are associated with the frequencies of wave groups occurring in irregular waves.

The first order wave forces and moments are the cause of the well known first order motions with wave frequencies. Due to the importance of the first order wave forces and motions they have been subject to investigation for several decades. As a result of investigations, methods have evolved by means of which these may be predicted with a reasonable degree of accuracy for many different vessel shapes.

For semi-submersibles which consist of a number of relatively slender elements such as columns, floaters and bracings, computation methods have been developed which determine the hydrodynamic loads on each element without taking into account interaction effects between the elements for the first order wave loads and motion problem this has been shown to give accurate results. See Hocft [1].

This paper deals with the mean and low frequency second order wave forces acting on stationary vessels in regular and irregular waves in general and, in particular, with a method to predict these forces on basis of computations.

The importance of the mean and low frequency wave drift forces from the point of view of motion behaviour and mooring loads on vessels moored at sea has been recognized only within the last few years. Verhagen and Van Sluijs [2], Hau and Blankarn [3] and Remery and Hermans [4] showed that the low frequency components of the wave drift forces in irregular waves could, even though relatively small in magnitude, excite large amplitude low frequency horizontal motions in moored structures. It was shown that in irregular waves the drift forces contain components with frequencies coinciding with the natural frequencies of the horizontal motions of moored vessels. Combined with the fact that the damping of low frequency horizontal motions of moored structures is generally very low, this leads to large amplitude resonant behaviour of the motions. See Fig. 1. Remery and Hermans [4] established that the low frequency components in the drift forces are associated with the frequencies of groups of waves present in an irregular wave train.

The vertical components of the second order forces are sometimes known as suction forces. This term is generally applied in connection with the mean wave induced vertical force and pitching moment acting on submarine vehicles when hovering or travelling near the free surface. It is shown by Bhattacharya [5] that in extreme cases the upward acting suction force due to waves can cause a submarine vehicle to rise and breach the surface, thus posing a problem concerning the control of the vehicle in the vertical plane.
The vertical components of the second order wave forces have also been connected with the phenomena of the steady tilt of semi-submersibles with low initial static stability as indicated by Kuo et al [6]. Depending on the frequency of the waves it has been found that the difference in the suction forces on the floaters of a semi-submersible can result in a tilting moment, which can cause the platform to tilt towards or away from the incoming waves. Such effects are of importance in judging the minimum static stability requirements for such platforms.

In order to determine the behaviour of moored structures in waves, model tests are often carried out. Simulation techniques based on numerical computations are becoming of increasing importance in the design phase of many floating structures however. For such simulation studies accurate numerical data on the behaviour of the mean and low frequency wave forces are desirable, so that meaningful results can be given regarding the systems under investigation. See for instance Van Oortmerssen [7] and Arai et al [8] and Wichers [9]. In order to produce numerical results, however, a reliable theory must be available on which calculations can be based.

Wahab [10] and Pijfjes and Brink [11] formulated expressions for the drift forces on semi-submersibles based on Morison's equation. The drift force on the total structure is assumed to be the sum of the forces acting on the columns and floaters which are determined without taking into account interaction effects. Such formulations indicate that the viscous drag term in Morison's equation can result in a mean drift force in regular waves. Furthermore, the drift force is shown to be a cubic function of the wave amplitude rather than a quadratic function as is predicted by potential theory.

Pinkster [12] computed the mean and low frequency second order wave drift forces acting on a semi-submersible in head waves based on potential theory methods which neglect the effects of viscosity. Comparison of results of computations with results of model tests indicated that such methods are capable of giving good quantitative predictions of drift forces on such structures. Since viscous effects are not included in potential theory methods this result indicates that wave drift forces cannot be predicted correctly by using methods based on Morison's equation.

In this paper computed and experimental results on the mean drift forces in regular waves and irregular waves on a semi-submersible will be presented. The computations are based on direct integration of second order pressure and force contributions acting on the instantaneous wetted part of the hull of a semi-submersible. The fluid motions and pressures are obtained using a three-dimensional diffraction program based on linear potential theory. Results of computations and model tests on mean horizontal drift forces in regular waves are compared for three wave directions, viz.: head waves, quartering waves and beam waves.

In order to show that drift forces also act in the vertical direction, results of computations and experiments for the same wave directions will also be compared with respect to the mean vertical drift force and trim moment in regular waves on a horizontal submerged cylinder. For the semi-submersible results of model tests in irregular head waves will also be compared with computations. Before treating results of model tests and computations a brief review of the computation method is given here. A complete review is given in ref. [13].

THEORY

The computations are based on potential theory. It is assumed that the fluid is inviscid, incompressible and irrotational. The fluid motions are described by a velocity potential $\phi$

$$\phi = \phi_1 + \phi_2 . . . . . . . . . . . . (1)$$

where:

$\phi_1$ = first order velocity potential from which first order pressures and forces are derived
$\phi_2$ = second order velocity potential
$\epsilon$ = a small parameter $< 1$

Both of these potentials may be written in the following form:

$$\phi = \phi_v + \phi_d + \phi_b . . . . . . . . . . . . (2)$$

in which:

$\phi_v$ = velocity potential of incoming waves
$\phi_d$ = velocity potential due to diffraction of $\phi_v$ on the stationary body
$\phi_b$ = velocity potential due to the motion of the body

The second order potentials $\phi_2$ and $\phi_2^d$ are quadratic functions of the first order potentials $\phi_1$, $\phi_1^d$ and $\phi_1^b$.

In developing the expressions for the wave drift forces and moments use is made of three systems of co-ordinate axes; see Fig. 2. The first is a fixed $X_0-X_1-X_2$ system with origin in the mean free surface. The potentials and the linear motions of the structure are defined relative to this system. The second is the body axes $0-X_1-X_2$ with origin in the centre of gravity (c) of the structure. The angular motions are defined relative to this system. The third is the $0-X_1-X_2$ system with origin in $0$ but with axes parallel to the $0-X_1-X_2$ system. Forces and moments will be defined relative to the $0-X_1-X_2$ system of axes. This system is chosen since we are generally interested in, for instance, horizontal forces rather than in forces along the body axes which are continually carrying out angular motions.

The hydrodynamic forces are obtained by integration of the pressure over the wetted part of the hull:

$$\mathbf{F} = -\iint p \mathbf{N} \mathbf{d}S . . . . . . . . . . . . . . . . . . . . . . . . . (3)$$

in which:

$p$ = pressure obtainable from Bernoulli's equation
According to Bernoulli's equation:
\[ p = p_0 - \rho g x - \rho \phi_t = \frac{1}{2} \rho \left| \nabla \phi \right|^2 + C(t) \]  
(4)
in which:
\[ p_0 = \text{atmospheric pressure} \]
\[ \rho = \text{mass density of the fluid} \]
\[ \phi = \text{velocity potential describing the fluid flow; see equation (1)} \]
\[ C(t) = \text{a constant depending only on time } t \]
\[ x_3 = \text{vertical position of point under consideration in } 0-X_1-X_2-X_3 \text{ system of co-ordinates} \]

In this equation \( p_0 \) and \( C(t) \) may be taken equal to zero without loss of generality.

In order to determine the second order wave drift forces we assume that the structure is carrying out small first order oscillatory motions about a mean position. The first order linear motions of a point on the hull of the structure relative to the 0-X_1-X_2-X_3 system is:
\[ \mathbf{e}^{(1)} = \mathbf{e}_x^{(1)} + \mathbf{e}_y^{(1)} + \mathbf{e}_z^{(1)} \times \mathbf{x} \]  
(5)
where:
\[ \mathbf{e}_x^{(1)} = \text{motion vector of } G \]
\[ \mathbf{e}_y^{(1)} = \text{angular motion vector with components } e_y^{(1)}, x_y^{(1)} \text{ and } y_y^{(1)} \]
\[ \mathbf{x} = \text{position vector of the point on the hull relative to the body axes } 0-X_1-X_2-X_3 \]

Since the structure is also carrying out first order angular motions, the normal vector \( \mathbf{n} \) of surface elements is varying relative to the 0-X_1-X_2-X_3 system of axes:
\[ \mathbf{n} = \mathbf{e}_x^{(1)} \times \mathbf{n} = 0 \]  
(6)
where:
\[ \mathbf{n} = \text{normal vector relative to the } 0-X_1-X_2-X_3 \text{ system of axes} \]

Taking into account that a point on the wetted surface is carrying out first order motions according to equation (5), the pressure in the point as given by Bernoulli's equation (4) can be expressed in terms of the pressure \( p_m \) in the mean position and the pressure gradient:
\[ p = p_m + \mathbf{e}^{(1)} \cdot \nabla \mathbf{p}_m \]  
(7)
Substitution of Bernoulli's equation leads to the following pressure:
\[ p = p_c + \varepsilon \phi^{(1)} + \varepsilon \phi^{(2)} \]  
(8)
where:
\[ p_c = \]  
(9)
\[ \phi^{(1)} = \]  
(10)
\[ \phi^{(2)} = \]  
(11)
in which:
\[ p_c = \text{constant part of the hydrostatic pressure} \]

Substitution of equations (9), (10), (11) and (6) in equation (3) and taking into account that the wetted surface \( S \) can be subdivided in a constant part \( S_0 \) and an oscillating part \( S_1 \) near the water line [see Fig. 2] finally leads to the following expression for the second order force vector:
\[ \mathbf{F}^{(2)} = \int \left( \mathbf{n} \cdot \mathbf{e}^{(1)} + \mathbf{e}^{(1)} \times \left( \mathbf{w} \cdot \mathbf{x}\left( 1 \right) \right) \right) dS \]  
(12)

Following a similar development the drift moment vector becomes:
\[ \mathbf{M}^{(2)} = \int \mathbf{g} \mathbf{e}^{(1)} \times \mathbf{x} dS \]  
(13)
in which:
\[ \mathbf{g} = \text{first order oscillatory relative wave elevation at the water line } \mathbf{w} \]
\[ dS = \text{length element measured along the mean water line} \]
\[ \mathbf{M} = \text{mass matrix} \]
\[ \mathbf{I} = \text{mass moment of inertia matrix} \]

Equations (12) and (13) express the forces and moments in the time domain.

It can be shown that these expressions can be used to formulate the drift forces and moments in the frequency domain in terms of quadratic transfer functions. These transfer functions depend on two frequencies rather than on one frequency as is the case with first order transfer functions. These transfer functions give the relationship between the drift forces and regular wave groups consisting of two regular waves with different frequencies. The quadratic transfer functions may be used to determine the mean drift forces and spectral density of the drift forces in regular waves or to predict the wave drift forces in time domain for arbitrary irregular waves. Given an irregular wave train of the following type:
\[ \xi(t) = \sum_{i=1}^{N} \xi_i \sin(\omega_i t + \phi_i) \]  
(14)
the low frequency part of the square of the wave elevation is:

\[ \zeta^2(t) = \sum_{i=1}^{N} \sum_{j=1}^{N} \zeta_i \zeta_j \cos(\omega_i - \omega_j)t + (\epsilon_i - \epsilon_j) \]  

(15)

In this wave train the wave drift force is found from:

\[ p_i(t) = \sum_{i=1}^{N} \sum_{j=1}^{N} \zeta_i \zeta_j \cos(\omega_i - \omega_j)t + (\epsilon_i - \epsilon_j) \]  

(16)

in which:

\[ p_{ij} = p_{ji} = \text{in-phase part of the quadratic transfer function for the drift force dependent on the frequencies } \omega_i \text{ and } \omega_j \]

\[ q_{ij} = -q_{ji} = \text{quadature part of the quadratic transfer function} \]

Equation (16) shows that the second order forces are closely related to the square of the wave elevation given in equation (15). The mean or constant part of the drift forces are found by putting \( \omega_i = \omega_j \) in equation (16). This becomes:

\[ p_{ij} = \sum_{i=1}^{N} \zeta_i^2 \]  

(17)

In irregular waves with the spectral density defined as:

\[ S_\zeta(\omega) = \frac{1}{\omega^2} \]  

(18)

equation (17), when written in the continuous form, becomes:

\[ p(\omega) = 2 \int_{0}^{\infty} S_\zeta(\omega) P(\omega,\omega) \, d\omega \]  

(19)

where \( P(\omega,\omega) \) is the equivalent of \( p_{ii} \).

The low frequency part of equation (16) may be expressed in terms of a force spectrum as follows:

\[ p_\zeta(\omega) = 8 \int_{0}^{\infty} S_\zeta(\omega) S_\zeta(\omega\mu) T^\zeta(\omega\mu,\omega) \, d\omega \]  

(20)

where:

\[ T^\zeta(\omega\mu,\omega) = T^\zeta(\omega\mu,\omega) + \xi^2(\omega\mu,\omega) \]  

(21)

in which \( P(\omega\mu,\omega) \) is the equivalent of \( p_{ij} \) for \( \omega_i = \omega_j = \mu \). The quadratic transfer functions may also be used to determine the wave drift forces in the time domain; see ref. [14].

In this paper attention will be focused on the frequency domain quadratic transfer functions only. Final evaluation of the quadratic transfer functions for the drift forces is possible through use of a three-dimensional diffraction theory computer program. By means of such computer programs it is possible to determine the first order velocity potentials and the first order vessel motions from which the drift forces are then derivable. A description of such computation methods is given in ref. [7] and in ref. [13].

**MODEL TESTS AND COMPUTATIONS**

In Table I the main particulars of the vessels are given, while the body plans are given in Fig. 3. The semi-submersible is a conventional six-column, two floater design, with circular columns and rectangular floaters. The submerged cylinder is circular in cross-section with hemispherical ends.

The model tests on the semi-submersible were carried out in the Wave and Current Laboratory of the Netherlands Ship Model Basin, Wageningen. This basin measures 60 x 40 x 1 metres. The model tests on the submerged cylinder were carried out in the Seakeeping Laboratory of the same institute. This basin measures 100 x 24 x 2.5 metres.

**TESTS IN REGULAR WAVES**

For the tests in regular waves, the models were restrained by soft linear spring mooring lines which incorporated force transducers to measure the mean drift forces. The spring constants of the soft mooring system were chosen so that no dynamic magnification of motions due to the restraining system occurred in the range of wave frequencies tested. Tests in regular waves were carried out for three wave directions, viz.: head waves (180 deg.), starboard bow quartering waves (135 deg.) and beam waves (90 deg.).

**COMPUTATIONS**

In order to compute the quadratic transfer functions for mean and low frequency wave drift forces using three-dimensional diffraction theory, the actual shape of the wetted part of the hull is approximated by a number of plane facets or panels. Each panel represents a pulsating source which contributes to the velocity potential describing the flow. The water line of the vessel is approximated by a number of line elements. This is necessary in order to evaluate the first part of the right-hand side of equation (12) and equation (13). The facet distribution of one side of the semi-submersible and the water line discretization of one column is shown in Fig. 4. The facet distribution of the cylinder is shown in Fig. 5. Computations on the mean drift forces were carried out for the same wave directions as applied in the model tests.

**RESULTS OF COMPUTATIONS AND MEASUREMENTS IN REGULAR WAVES**

The mean longitudinal and transverse drift forces and the mean yaw moment on the semi-submersible are given in Fig. 6, 7 and 8. The mean vertical drift force and mean trim moment on the cylinder are given in Fig. 9. All results are given in non-dimensional form. The non-dimensional drift forces and moments correspond with the non-dimensional form of the quadratic transfer function \( P_{ij} \). Since both frequencies on which the transfer functions depend are equal in regular waves, the transfer functions are for this case dependent on one frequency, being in this case the frequency of the regular wave.
**COMPARISON OF RESULTS OF COMPUTATIONS AND MEASUREMENTS IN REGULAR WAVES**

From the comparison of the results on the mean drift forces and yaw moment on the semi-submersible, shown in Figs. 6 through 8, it can be concluded that the drift forces are also predicted correctly. With respect to the results obtained for the semi-submersible it is noted that the mean drift forces given in Figs. 6 through 8 show rapid fluctuations with the wave frequency for all wave directions. These fluctuations in the mean drift forces are related to interaction effects between the columns. In head waves the results given in Fig. 6 show a marked reduction in the mean drift force at a non-dimensional wave frequency of 2.2. In beam waves the results given in Fig. 8 show a similar reduction at a non-dimensional wave frequency of about 1.8. The wave lengths corresponding to these frequencies for the head waves and the beam wave case amount to 43 m and 62 m respectively for the vessel size as given in Fig. 3. These values are quite close to the distance between the columns as measured in the direction of the wave propagation which amount to 38 m and 60 m respectively. In such cases standing wave effects occur between the columns.

In order to check the quadratic relationship between the mean second order forces and the wave amplitude, experiments with the semi-submersible were carried out in waves with different amplitudes. In general the quadratic relationship between the mean forces and the wave amplitude is confirmed with a reasonable degree.

The vertical drift force and trim moment on the submerged horizontal cylinder given in Fig. 9 show that these quantities are also predicted correctly by potential theory methods.

In order to illustrate the importance of the several components of the drift forces given in equation (12), a breakdown of the mean longitudinal drift force in head waves on the semi-submersible and the mean vertical force on the cylinder in head and in beam waves is given in Fig. 10 and Fig. 11 respectively. The numerals in these figures refer to the following components of equation (12):

<table>
<thead>
<tr>
<th>Component</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>I: First order relative wave elevation</td>
<td>[- \frac{1}{2} \rho g \int (\zeta_T^1)^2 , n , ds ]</td>
</tr>
<tr>
<td>II: Pressure drop due to first order velocity</td>
<td>[- \int \rho \nabla \cdot \bar{V}_t \cdot (\nabla \cdot \bar{V}_t) , n , ds ]</td>
</tr>
<tr>
<td>III: Pressure due to product of gradient of first order pressure and first order motion</td>
<td>[- \int \rho \bar{V}_t \cdot \nabla \bar{V}_t \cdot \nabla \bar{V}_t , n , ds ]</td>
</tr>
<tr>
<td>IV: Contribution due to products of first order angular motions and inertia forces</td>
<td>[- \alpha(1) \times (M_{I1}(1)) ]</td>
</tr>
</tbody>
</table>

**TRENDS IN IRREGULAR WAVES**

One test was carried out with the semi-submersible in irregular head waves. The wave spectrum is given in Fig. 12. The purpose of this test was to measure directly the mean and low frequency longitudinal wave drift force in a realistic sea condition. The measured drift force record was analysed by means of cross-bispectral methods (see ref. [13]) which yield the results in estimates of the quadratic transfer function \( T_{12} \) for regular waves and of the value of the amplitude \( T_{12} \) of the quadratic transfer function in regular wave groups consisting of two irregular waves with frequencies \( \omega_1 \) and \( \omega_2 \). These results are compared with results of computations based on potential theory.

The measuring system used for these tests is described extensively in ref. [13] and will be left out of consideration here.

The results of computations and measurements are compared in Fig. 13. The results show the value of the quadratic transfer function \( T_{12} \) to a base of full scale mean frequency of the irregular wave components for the case that \( \omega_1 = \omega_2 \), which is equivalent to the mean force in regular waves, and for the case that \( \omega_1 \neq \omega_2 \). In this case the wave group frequency and hence the frequency of the second order force equals 0.1 rad./sec. full scale. The experimental data is denoted by C.B.S. (cross-bispectral analysis) results obtained from measured data. In the Fig. 13(b) two computed lines are shown denoted by 'total' and computed without \( \phi(2) \). The computation is based on equation (12) excluding the contribution due to the second order potentials \( \phi(2) \) and \( \bar{\phi}(2) \).

Comparison of results of computations and measurements shows that also results of tests in realistic irregular waves tend to confirm that potential theory predicts wave drift forces with reasonable accuracy.
The agreement obtained between results of model tests and computations based on potential theory indicates that viscous effects on wave drift forces on semi-submersibles are small in conditions of waves only. Not only the horizontal drift forces are reasonably well predicted using potential theory but also the vertical drift forces.

In conditions of waves combined with current viscous effects probably do play a role with respect to the mean and low frequency forces however. This is deduced from results of model tests, which indicate that the total mean force in conditions of waves and current is not always equal to the sum of the mean forces found in waves only and in current only. Computations based on Morison's equation carried out by Pijpers and Brink [11] and Ferretti and Berta [15] confirm this effect in qualitative sense. This suggests that there are interaction effects between current and waves which, up to the present, have not been properly quantified. It has been suggested that wave drift forces are significantly changed due to the presence of current. On the other hand it is also possible that the presence of waves disturbs the current flow pattern considerably relative to the still water case, thus leading to a change in the current drag. More research will be needed in order to clarify this situation.

NOMENCLATURE

\( f_1^{(2)}, f_2^{(2)}, f_3^{(2)} \) mean second order drift force in regular waves in longitudinal, transverse and vertical directions respectively as given in figures

\( m_1^{(2)}, m_2^{(2)}, m_3^{(2)} \) mean second order roll, trim and yaw moments respectively due to drift forces in regular waves

\( \mu \) in figures, wave direction

\( \zeta_a^{(1)} \) amplitude of a regular wave

\( V \) displaced volume of the structure

\( g \) acceleration of gravity

\( \zeta_{1/3} \) significant wave height

\( T \) mean wave period

REFERENCES

TABLE I

<table>
<thead>
<tr>
<th>Designation</th>
<th>Symbol</th>
<th>Unit</th>
<th>Semi-submersible</th>
<th>Submerged horizontal cylinder</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length (between perpendiculars)</td>
<td>$L_{PP}$</td>
<td>m</td>
<td>100.00</td>
<td>75.60</td>
</tr>
<tr>
<td>Breadth</td>
<td>$B$</td>
<td>m</td>
<td>76.00</td>
<td>8.40</td>
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<tr>
<td>Draft</td>
<td>$T$</td>
<td>m</td>
<td>20.00</td>
<td>12.60*</td>
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<tr>
<td>Displacement volume</td>
<td>$V$</td>
<td>m$^3$</td>
<td>35,925</td>
<td>4,034</td>
</tr>
<tr>
<td>Water depth</td>
<td>$W_d$</td>
<td>m</td>
<td>40.0</td>
<td>75.0</td>
</tr>
</tbody>
</table>

* Distance between base and mean still water surface

Main particulars of the semi-submersible and the horizontal submerged cylinder.

Fig. 1 - Surge motions of a semi-submersible in irregular head waves.

Fig. 2 - Systems of co-ordinate axes.
**Fig. 3 - The vessels.**

**Facet Schematisation Semi-Submersible**
- Total 216 facets

**Facet Schematisation Cylinder**
- Total 286 facets

**Water Line Schematisation**
- Total 72 elements

**Fig. 4 - Facet and water line element distribution of the semi-submersible (only one floater shown).**

**Fig. 5 - Facet distribution of the submerged horizontal cylinder.**

**Fig. 6 - Mean longitudinal drift force in regular head waves (180°) on the semi-submersible.**
Fig. 7 - Mean longitudinal and transverse drift forces and mean yaw moment on the semi-submersible in regular bow quartering waves (135°).

Fig. 8 - Mean transverse drift force in regular beam waves (90°) on the semi-submersible.
Fig. 9 - Mean vertical drift force and trim moment on the cylinder in regular head waves, bow quartering waves, and beam waves.

Fig. 10 - Components of the mean longitudinal drift force in regular waves on the semi-submersible.

Fig. 11 - Components of the mean vertical drift force in regular head and beam waves on the cylinder.
Fig. 12 - Spectrum of irregular waves.

Fig. 13 - Quadratic transfer function of the longitudinal drift force on the semi-submersible in regular waves and regular wave groups.