DYNAMIC BEHAVIOUR OF MOORING LINES

H.J.J. van den Boom

Maritime Research Institute Netherlands, P.O. Box 28, 6700 AA Wageningen

SUMMARY

In designing offshore mooring systems the dynamic behaviour of mooring chains, wires and multi-component lines is of increasing importance. Various authors have reported on experimental results and numerical techniques related to this subject.

An extensive research program has been carried out to gain further insight in the mechanism of the dynamic behaviour of mooring lines, to quantify the effects of important parameters, with special attention to the maximum tension, and to validate a numerical model. For this purpose, results of a developed computer algorithm based on the Lumped Mass Method were compared with results of harmonic oscillation tests for various lines and water depths at different model scales. The ultimate validation was carried out by comparing tension records from irregular wave model tests with those obtained from numerical analysis using the measured fairlead motion as input.

Results from this study clearly show the importance of dynamic analysis for various mooring configurations. Dynamic tension amplification is strongly influenced by non-linearities due to catenary effects, elasticity and drag. The lumped mass algorithm presented has been proven to be an effective tool to quantify the dynamic behaviour of multi-component mooring configurations. Dynamic tensions in mooring systems may affect the low frequency motions of the moored structure.

1. INTRODUCTION

The increasing application of large moored and guyed offshore structures has put high demands upon the design of mooring arrangements. Important parameters in this respect are the large displacement of the structure, deep and hostile waters and the required round-the-year workability. The wide variety of mooring systems may be illustrated by the existence of shallow and deep water single point moorings with temporarily or permanently moored tankers, clump weight systems used for guyed towers and wire moorings of semi-submersible crane vessels.

Current design procedures comprise dynamic motion analysis of the moored structure and computations of mooring line tension based on the extreme position of the vessel and the static load-excursion characteristics of the mooring system. In this so-called quasi-static mooring analysis all other phenomena affecting the maximum line tension are accounted for by an overall safety factor as required by classification and regulatory authorities. A typical value of this safety factor is 3 for operational conditions and 2 for survival conditions.

From both theoretical and experimental research it is known that the dynamic behaviour of a mooring line induced by high frequency oscillations of the upper-end may contribute significantly to line tensions and motions. Therefore these dynamic effects may be of importance in the design of the mooring arrangement. In some cases mooring line dynamics might also affect the motions of the moored object.

In the past decades various authors have reported on experimental results and numerical techniques regarding above aspects. Amongst others Van Sluijs and Blok, [1], found from systematical series of forced oscillation model tests that the ratios of maximum dynamic tension and maximum quasi-static tension strongly depend on the frequency of oscillation. This dynamic ratio increased with increasing oscillation amplitude and pre-tension and with reduction of line mass.

Traditional theoretical approaches to solve the dynamic behaviour of cable systems were based on semi-analytical techniques. The obstacles for a pure analytical approach caused by geometric non-linearities were removed in order to reduce the equations of motion to ordinary differential equations. Other approaches such as the perturbation
techniques derived linear equations of motion by evaluating small variations about an equilibrium configuration.

Application of chains and cables in various underwater systems required more general approaches to the problem. It was found by assuming the line to be composed of an interconnected set of discrete elements that the system of partial differential equations describing the variables along the line could be replaced by equations of motion in an earth-bound system of co-ordinates. The most successful discrete element techniques, the "lumped parameter method", better known as the Lumped Mass Method (LMM) and the Finite Element Method (FEM) will be discussed here briefly.

Lumped Mass Method

This technique involves the lumping of all effects of mass, external forces and internal reactions at a finite number of points ("nodes") along the line. By applying the equations of dynamic equilibrium and continuity (stress/strain) to each mass a set of discrete equations of motion is derived. These equations may be solved in the time domain directly using finite difference techniques. Material damping, bending and torsional moments are normally neglected. This procedure implies that the behaviour of a continuous line is modelled as a set of concentrated masses connected by massless springs.

Walton and Polacheck, [2], were the first authors who suggested this method to solve mooring problems caused by transient motions of a moored vessel. Their spacewise discretization neglected material elasticity. Moreover no data on fluid reactive forces were available and no validation of the algorithm was given. The explicit central difference method was proven to provide conditionally stable solutions for the given schematization.

In recent years the LMM has been developed further in order to solve offshore mooring problems (Wilhelmy et al. [3], [4]). Nakajima, Motora and Fujino, [5], extended the model of Walton and Polacheck with material elasticity and sea floor contact. Using hydrodynamic force coefficients derived from forced oscillation tests on model chains, they found a good agreement between numerical results and model tests for harmonic oscillation. Unfortunately no slack conditions were investigated.

Finite Element Method

The Finite Element Method utilizes interpolation functions to describe the behaviour of a given variable internal to the element in terms of the displacements of the nodes defining the element (or other generalized co-ordinates). The equations of motion for a single element are obtained by applying the interpolation function to kinematic relations (strain/displacement), constitutive relations (stress/strain) and the equations of dynamic equilibrium. The solution procedure is similar to the LMM.

Various models based on the FEM have been presented either using linear or higher order shape functions. Recently FEM models for mooring line analysis were developed by Fylling and Wold, [6], and Larsen and Fylling, [7], and Lindahl and Sjöberg, [8]. Computer codes based on the FEM have proven to be less computer time efficient when compared with the LMM algorithms.

Unfortunately the validity of the presented numerical models has so far not been demonstrated clearly. Nor were the effects of the type of line, the water depth and the upper-end oscillation quantified systematically. Therefore an extensive research program on mooring line dynamics has been carried out by the N.S.M.B. Laboratories of MARIN as part of the Netherlands Marine Technological Research (MaTS) program. The project was sponsored by the following parties:

Dutch Ministry of Economic Affairs
Gusto Engineering
Heerema Engineering Service
MARIN
Shell Internationale Petroleum Maatschappij
Van Rietschoten & Houwens

2. NUMERICAL MODEL
2.1 Problem definition

A mooring line connected to a structure floating in irregular waves, wind and current is subjected to line-end loads, weight, buoyancy, sea floor reaction forces, line inertia and fluid loading. Bearing in mind the large mass of the structure it may be assumed that the motions of the structure in the region of wave frequencies are not affected by the mooring line tension. On the other hand the dynamic response of the line will cover this frequency region. Hence it may be assumed that the analysis of the wave frequency motions of the structure and the behaviour of the mooring line can be treated separately. The fairlead motion is thus the boundary condition for the line motions.

The fluid loading of the line is due to wave induced orbital velocities, current and line motions and may be divided in components proportional to the relative fluid acceleration ("added inertia") and those proportional to the relative velocity squared ("drag"). Wave contributions to the relative velocity are normally small and neglected here.
When deriving the equations of motion for a mooring line it is preferable to describe the fluid loading in components along the line (tangential) and in transverse (normal) direction. Taking into account the catenary shape and allowing large deflections of the line, this means that the fluid loading has to be defined in a local system of co-ordinates while the ultimate motions are required in an earth-fixed "global" system of co-ordinates.

2.2 Algorithm

The mathematical model chosen is a modification of the LMM as presented by Nakajima et al., [5]. A computer program, named DYNLINE, applies this method in two dimensions assuming that the mooring line remains in the vertical plane through both line ends.

![Diagram of mooring line discretization](image)

**Fig. 1. Discretization of mooring line by a lumped mass method**

The spacewise discretization of the mooring line is obtained by lumping all forces to a finite number of nodes ("lumped-masses"). The finite segments connecting the nodes are considered as massless springs accounting for the tangential elasticity of the line. The line is assumed to be fully flexible in bending directions. The hydrodynamic forces are defined in the local system of co-ordinates (tangential and normal direction) at each mass.

In order to derive the governing equations of motion for the j-th lumped-mass, Newton's law is written in global co-ordinates:

\[ [M_j] + [m_j(\tau)] \ddot{x}_j(\tau) = F_j(\tau) \quad \ldots (1) \]

where:

- \([M_j]\) = inertia matrix
- \([m_j(\tau)]\) = added inertia matrix
- \(\tau\) = time
- \(x_j\) = displacement vector
- \(F_j\) = external force vector.

The added inertia matrix can be derived from the normal and tangential fluid forces by directional transformations:

\[ [m_j(\tau)] = a_{nj} [A_{nj}(\tau)] + a_{tj} [A_{tj}(\tau)] \quad \ldots (2) \]

where \(a_{nj}\) and \(a_{tj}\) represent the normal and tangential added mass:

\[ a_{nj} = \rho C_{In} \pi/4 D_j^2 \quad \ldots \ldots (3) \]

\[ a_{tj} = \rho C_{It} \pi/4 D_j^2 \quad \ldots \ldots (4) \]

\([A_{nj}]\) and \([A_{tj}]\) are directional matrices given below for the two-dimensional case:

\[ [A_{nj}] = \begin{pmatrix} \sin \phi_j & -\sin \phi_j \cos \phi_j \\ -\sin \phi_j \cos \phi_j & \cos^2 \phi_j \end{pmatrix} \quad (5) \]

\[ [A_{tj}] = \begin{pmatrix} \cos \phi_j & \sin \phi_j \cos \phi_j \\ -\sin \phi_j \cos \phi_j & \sin^2 \phi_j \end{pmatrix} \quad (6) \]

\[ \phi_j = (\phi_j + \phi_{j-1})/2 \]

In order to derive the governing equations of motion for the j-th lumped-mass, Newton's law is written in global co-ordinates:

\[ [M_j] + [m_j(\tau)] \ddot{x}_j(\tau) = F_j(\tau) \quad \ldots (1) \]

where:

\[ [M_j]\] = inertia matrix

\[ [m_j(\tau)]\] = added inertia matrix

\(\tau\) = time

\(x_j\) = displacement vector

\(F_j\) = external force vector.

The added inertia matrix can be derived from the normal and tangential fluid forces by directional transformations:

\[ [m_j(\tau)] = a_{nj} [A_{nj}(\tau)] + a_{tj} [A_{tj}(\tau)] \quad \ldots (2) \]

where \(a_{nj}\) and \(a_{tj}\) represent the normal and tangential added mass:

\[ a_{nj} = \rho C_{In} \pi/4 D_j^2 \quad \ldots \ldots (3) \]

\[ a_{tj} = \rho C_{It} \pi/4 D_j^2 \quad \ldots \ldots (4) \]

\([A_{nj}]\) and \([A_{tj}]\) are directional matrices given below for the two-dimensional case:

\[ [A_{nj}] = \begin{pmatrix} \sin \phi_j & -\sin \phi_j \cos \phi_j \\ -\sin \phi_j \cos \phi_j & \cos^2 \phi_j \end{pmatrix} \quad (5) \]

\[ [A_{tj}] = \begin{pmatrix} \cos \phi_j & \sin \phi_j \cos \phi_j \\ -\sin \phi_j \cos \phi_j & \sin^2 \phi_j \end{pmatrix} \quad (6) \]

\[ \phi_j = (\phi_j + \phi_{j-1})/2 \]

The nodal force vector \(F\) contains contributions from the segment tension \(T\), the drag force \(FD\), buoyancy and weight \(FW\) and soil forces \(FS\):

\[ F_j(\tau) = T_j(\tau) \Delta x_j(\tau) - T_{j-1}(\tau) \Delta x_{j-1}(\tau) + FD_j(\tau) + FW_j + FS_j(\tau) \quad \ldots (7) \]

where:

\(\Delta x_j\) = the segment basis vector

\((x_{j+1} - x_j)/\varepsilon_j\)

\(\varepsilon_j\) = original segment length.
The drag force may be derived from the normal and tangential force components:

\[ \mathbf{f_D} = \mathbf{f}_n + \mathbf{f}_t \]

where:

\[ f_{D} \] = drag force in local coordinates
\[ U_{J} \] = relative fluid velocity in local coordinates
\[ c_{J} \] = current vector in global coordinates
\[ p \] = fluid specific density
\[ D \] = characteristic segment diameter
\[ \alpha \] = segment length
\[ C_{Dn}, C_{Dt} \] = normal and tangential drag coefficients.

The directional matrices \([\mathbf{R}_J]\) and \([\mathbf{u}_J]\) are used to transform the global velocities into local velocities (11) and the local drag force components into global forces (8) respectively.

\[ \mathbf{[u}_J = (1 - \mathbf{R}_J) (\mathbf{u}_J) \]

The fluid reactive force coefficients \(a_n, a_t, C_{Dn}, \text{and} C_{Dt}\) were derived from forced oscillation tests and free hanging towing tests with model chain and wire sections.

The volumetric diameter defined by equation (14) proved to be an accurate parameter in the dimensionless hydrodynamic coefficients:

\[ d_{c} = 2\sqrt{V/\pi \varepsilon} \]

where:

\(V\) = segment volume
\(\varepsilon\) = segment length.

From the model tests it was concluded that frequency independent coefficients can be used for normal mooring chains and wires.

Sea floor contact may be simulated by spring-damped systems. Tangential soil friction forces may be of importance when the line part on bottom is extremely long. Transverse soil reactive forces may be of importance for 3-D problems. Both effects are neglected here.

\[ F_{SJ}(3) = -c_{J}x_{J}(3) + b_{J} \dot{x}_{J}(3) \quad x_{J}(3) \leq 0 \]
\[ F_{SJ}(3) = 0.0 \quad x_{J}(3) > 0 \]

The time domain relations between nodal displacements, velocities and accelerations may be approximated by finite difference methods such as the Houbolt scheme, [9]:

\[ \mathbf{x}_{J}(t+\Delta t) = \frac{1}{6\Delta t} \left\{ 11x_{J}(t+\Delta t) - 18x_{J}(t) + 9x_{J}(t-\Delta t) - 2x_{J}(t-2\Delta t) \right\} \quad \ldots \quad (16) \]

\[ \mathbf{x}_{J}(t+\Delta t) = \frac{1}{4\Delta t} \left\{ 2x_{J}(t+\Delta t) - 5x_{J}(t) + 4x_{J}(t-\Delta t) - x_{J}(t-2\Delta t) \right\} \quad \ldots \quad (17) \]

or:

\[ x_{J}(t+\Delta t) = \frac{5}{2} x_{J}(t) - 2x_{J}(t-\Delta t) + \frac{1}{2} x_{J}(t-2\Delta t) + \frac{1}{2} \Delta t^2 \ddot{x}_{J}(t+\Delta t) \quad \ldots \quad (17) \]

The segment tension \(T_{J}(t+\Delta t)\) is derived from the node positions by a Newton-Raphson iteration using the additional constraint equation for the constitutive stress-strain relation.

\[ \psi_{J}(t) = \mathbf{t}_{k}^T \mathbf{[\Delta \psi]}(t) \mathbf{[\Delta \psi]}(t)^{-1} \mathbf{[\Delta \psi]}(t) \quad \ldots \quad (19) \]

where:

\(\psi\) = segment length error vector
\(T_{k}\) = tentative segment tension vector at the \(k\)-th iteration \(T_{k}, T_{k-1}, \ldots, T_{k}\)
\(\Delta \psi\) = length error derivative matrix \([\partial \psi / \partial T]\)

For each time step the system of equation (19) should be solved until acceptable convergence of \(T_{k}(t+\Delta t)\) is obtained. The initial tentative tension can be taken equal to the tension in the previous step. Each node \(j\) is connected to the adjacent nodes \(j-1\) and \(j+1\), hence equation (19) represents a tridiagonal \((N\times3)\) system. Such equations may be efficiently solved by the so-called Thomas algorithm.

2.3 Computational procedure

The computational procedure followed by DYNLINE is illustrated by Fig. 3. In order to avoid instability and transient behaviour the simulation is started from an arbitrarily chosen state of equilibrium of the line. This can be the quasi-static condition of the mooring line found from catenary calculations or numerical integration methods, [10].

The simulation is initiated by applying a starting function to the upper-end boundary condition:

\[ x_{N}(t) = \frac{X_{N}(t)}{\cosh (4.0 t/T_{INF})} \quad \ldots \quad (20) \]

where:

\(T_{INF}\) = starting time.
The normal stiffness is non-linearly dependent on the normal displacement \( \delta_n \). For small deflections this stiffness equals:
\[
C_n = EA \frac{\delta_n}{s^2} \quad \text{............... (22)}
\]

Neglecting the damping the resonance frequencies of these parasitical motions may be approximated by:
\[
\omega_n = \sqrt{C(M + m)} \quad \text{............... (23)}
\]

For the usual types of mooring lines resonant response of separate masses in the lumped parameter model will not provide significant parasitical motions. This even holds true for clusters of masses. The occurrence of such may be prevented by increasing the number of nodes thus reducing the nodal mass and element length.

3. VALIDATION STUDY
3.1 Harmonic oscillation tests

Model tests utilizing harmonic upper-end forced oscillations of the line at five frequencies for eleven combinations. The water depths ranged from 75 m to 608 m. Chains, steel wires and chain-wire combi-lines were investigated. For these tests, which were carried out according to Froude's law of similitude, use was made of steel studless chain and wire. The scale ratios ranged from 19 to 76. It should be noted that the chain links of the 1.0 and 2.0 mm chain were cut at one side. The \( EA \)-values were derived from tension-elongation tests.

The oscillation tests were carried out in the 220 m x 4 m x 4 m and the 240 m x 18 m x 8 m basins of N.S.M.B. During the tests the forced oscillation, generated by means of a mechanical large stroke oscillator, was measured by means of a potentiometer. The upper-end line tension and vertical angle were measured by means of a two-component force transducer while the tension at the anchor point was measured by means of a ring-type force transducer. The motions of the line were recorded by underwater video. The measured tensions were directly compared with the DYNLINE results. Moreover comparisons were carried out on the basis of the Dynamic Tension Amplification (DTA) defined as amplification of the maximum total quasi-static tension, i.e. the static tension at the maximum excursion. Fig. 4 shows the static load-excursion characteristics, the numerical discretization and the dynamic tension amplification for 152 mm chain at 150 m water depth. For a 76 mm chain-wire combi-line in 608 m water depth these results are presented in Fig. 5.
3.2 Irregular wave tests

Because of the non-linear phenomena involved, the ultimate validation of the developed computer program was carried out by means of model tests in irregular waves. A model of a floating structure was moored by means of two parallel lines and a tensioning weight as shown by Fig. 6.

Fig. 6. Test set-up for irregular wave tests

During the tests the motions of the structure were measured by means of an optical tracking device while the upper-end mooring line tensions and angles were measured by means of two-component strain-gauges. The fairlead motions derived from the measured motion at deck level were used as input to DYNLINE. This procedure enabled a deterministical comparison between experimental and numeric tension records.

Results for a 46,000 tons semi-submersible floating in irregular waves with a significant height of 13.0 m and a mean period of 15.5 s (Fig. 7) are given in Fig. 8 and Fig. 9. In order to show the contribution of the dynamic behaviour the computer simulations were repeated for 80 per cent reduced line diameters thus reducing drag (80%) and added inertia (96%).
4. DISCUSSION

Results of the present study clearly show that in practical situations the dynamic behaviour may contribute to the maximum tension significantly. Important parameters are the non-linear static load-excitation, the low frequency ("pre-") tension and the amplitude and frequency of the exciting upper-end oscillation.

The prime dynamic tension increase originated from the normal drag forces related to large global (first mode) line motions at the middle sections. Long periods of slackness even at low frequencies of oscillation occurred due to "flying" of the line under the influence of gravity and drag only. With increasing frequency the drag and inertia equalled gravity forces resulting in an "elevated equilibrum" of the line and normal motions in the upper section yielding lower DTA-values.

Inertia became of importance at higher wave frequencies especially for steel wires and multi-component lines.

Fig. 10 compares the dynamic tension excursion relation with the static characteristics. From this figure it appears that the maximum dynamic tension may be approximated by applying the material elasticity to the high frequency oscillations directly. For oscillations covering the taut situation of the line, however, the dynamic amplification is small.

A good correlation between measured and calculated line tensions was found during the harmonic oscillation tests for the wide range of situations investigated. Parasitical normal line motions originating from the numerical discretization may result in high frequency secondary tension components especially when the normal drag of the line is small. Due to the high frequencies and the small magnitude of the tension components these numerical modelling effects are of minor importance for engineering applications. Dynamic sea floor reaction forces do not affect the behaviour of the line and can be modelled as critical damped springs to prevent numerical instabilities. Deterministic correlations for irregular upper-end oscillations have clearly shown the strong increase of line tension also in practical situations due to the dynamic behaviour of the line. By means of the "reduced dynamics" simulation it was found that the normal drag forces govern the mechanism of dynamic motion of tension amplification of the line. As shown by Figs. 8 and 9 the high frequency secondary tension components which result from the numerical discretization of the line are of minor importance. Both harmonic and irregular upper-end correlation tests have clearly shown the validity and applicability of the presented numerical model. It may therefore be concluded that the use of the Lumped Mass Method does provide efficient and accurate predictions of dynamic motions and tensions for offshore mooring analysis.

The first assumption in the dynamic analysis of mooring lines, viz. the separation of motion analysis of the floating structure and the dynamic upper-end line tension has not been discussed yet. Observing typical regions of response at wave frequencies and resonant low frequencies of the moored structure the following interferences should be considered:

- dynamic tensions and motions of the structure at wave frequency;
- dynamic tensions and motions of the structure in the low frequency region;
- dynamic tensions in the wave frequency region and low frequency motions of the structure.

The first two interferences strongly depend on the geometry of the floating structure. Due to the large displacement the wave frequency motions will normally not be affected by the mooring forces. On the other hand the mooring system will respond quasi-statically to low frequency motions of the structure.

The third interference, viz. the effect of high frequency dynamic line tensions on the low frequency behaviour of the vessel, was investigated by means of several additional simulations for bi-harmonic oscillations. A typical low frequency oscillation with a period of 100 s and 10 m amplitude was combined with a 4 m wave frequency oscillation.

Fig. 7. Wave spectrum
Fig. 8. Correlation Lumped Mass Method - model test for 0.076 m chain at 292 m water depth

Fig. 9. Correlation Lumped Mass Method - model test for 0.076 m steel wire at 292 m water depth
Fig. 10. Dynamic tension-displacement relation

<table>
<thead>
<tr>
<th>( \omega ) (rad/s)</th>
<th>( X_a ) (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.063</td>
<td>10.0</td>
</tr>
<tr>
<td>0.500</td>
<td>4.0</td>
</tr>
</tbody>
</table>

Fig. 11. Effect of dynamic behaviour of mooring line on low frequency restoring forces
The low frequency energy in the bi-harmonic result was studied by removing the high frequency tension components by means of low-pass filtering. This result was compared with the tension due to the low frequency oscillation only. Time histories of such a comparison are given in Fig. 11.

The change of restoring forces experienced by the floating structure is illustrated by an increase in amplitude of low frequency tension and a phase shift. Dividing the tension record in a in-phase and quadrature phase component, it is clear that the dynamic behaviour of the mooring line may increase both the effective mooring stiffness and the low frequency damping. The latter can be of the same order of magnitude as the potential and viscous fluid damping acting on the vessel's hull directly and is therefore important for the low frequency behaviour of the moored structure.

5. CONCLUSIONS

The following major conclusions were drawn from the research program presented in this paper:

- The dynamic behaviour of mooring lines occurs in many practical offshore mooring situations and strongly increases the maximum line tensions.
- The use of the Lumped Mass Method does provide efficient and accurate predictions of dynamic line motions and tensions certainly for engineering application.
- The dynamic components of mooring line tension may affect the low frequency motions of the moored structure by increase of the virtual stiffness and damping of the system.

ACKNOWLEDGEMENT

The author is indebted to the sponsors of the research program "Mooring Line Dynamics" of the Netherlands Marine Technological Research (MaTS) for their kind permission to make use of the results from this program.

REFERENCES


